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ABSTRACTS

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Invariant toroidal manifolds of a certain class of discontinuous dynamical systems

F .A.Asrorov [1]

[1]Taras Shevchenko National University of Kyiv, Kyiv, Ukraine

asrorovfa@knu.ua

This paper studies a class of autonomous differential equations subjected to impulsive effects. Unlike the systems considered in previous works, where impulses were applied at fixed moments in time, here the system experiences impulses when the angular variable passes through a fixed set on the torus. In contrast to earlier assumptions, where the angular variable was not affected by impulses during its evolution, this restriction is lifted in the present study.

The system of differential equations under impulsive effects is considered. Initially, the behavior of phase curves of the discontinuous dynamical system on the torus is examined, and conditions are identified under which these phase curves densely cover the torus. Then, sufficient conditions for the existence of an asymptotically stable invariant set of the impulsive differential system are provided.

Derivations of Lie Algebras of Vector Fields in Infinitely Many Variables

Oksana Bezushchak[1]

[1]Taras Shevchenko National University of Kyiv, Kyiv, Ukraine

bezushchak@knu.ua

Let $\mathbb{F}[X_n]$, $n \geqslant 1$, be the polynomial algebra on the set of variables $X_n = \{x_1, \ldots, x_n\}$; and let $\mathbb{F}[X]$ be the polynomial algebra on the set of variables $X = \bigcup_{n \geqslant 1} X_n, .$

Consider the Lie algebra $W_n(\mathbb{F}) = \text{Der } \mathbb{F}[X_n]$ of all derivations of the polynomial algebra $\mathbb{F}[X_n]$. It is an infinite-dimensional simple Lie algebra of Cartan type (see [1]). F.Takens [2] proved that, over a field of characteristic 0, every derivation of $W_n(\mathbb{F})$ is inner.

Let as denote by $W_X(\mathbb{F}) = \text{Der } \mathbb{F}[X]$ the Lie algebra of all derivations of the polynomial algebra $\mathbb{F}[X]$.

The Lie algebra

$$W_\infty(\mathbb{F}) = \bigcup_{n \geqslant 1} W_n(\mathbb{F})$$

is a subalgebra of the algebra $W_X(\mathbb{F})$.

Theorem 1. *All derivations of the Lie algebra $W_X(\mathbb{F})$ over a field of zero characteristic are inner.*

Theorem 2. *Every derivation of the algebra $W_\infty(\mathbb{F})$ over a field of zero characteristic is a restriction of an adjoint operator $ad(a) : x \to [x, a]$, where the derivation a is an infinite sum*

$$\sum_{i=1}^{\infty} f_i(x_1, \ldots) \frac{\partial}{\partial x_i}$$

such that each variable $x_n \in X$ occurs in only finitely many polynomials f_i, $i \geqslant 1$.

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Ramanujan subshifts

I. V. Bondarenko[1]
[1]Taras Shevchenko National University of Kyiv, Kyiv, Ukraine
ievgen.bondarenko@knu.ua

A finite, connected, d-regular graph G is called a Ramanujan graph if every eigenvalue λ of its adjacency matrix satisfies either $\lambda = \pm d$ or $|\lambda| \leqslant 2\sqrt{d-1}$. The term "Ramanujan" comes from the RamanujanPetersson conjecture, whose proof for certain automorphic forms enabled the explicit construction of infinite families of Ramanujan graphs when $d = q + 1$ for a prime power q. Ramanujan graphs are optimal expanders (as the number of vertices tends to infinity), which have many applications in computer science. Furthermore, the Ihara zeta function of a regular graph satisfies the Riemann hypothesis if and only if the graph is Ramanujan.

In this talk, I will describe a dynamical interpretation of the Ramanujan property in terms of associated subshifts of finite type, extend this concept to define Ramanujan subshifts as subshifts of finite type with optimal mixing properties, and discuss their existence. Several open problems will be presented. The results discussed in this talk are based on a joint work with R. Grigorchuk and A. Vdovina.

Polynomial similarity of pairs of matrices

V. M. Bondarenko[1], **A. P. Petravchuk**[2]
[1]Institute of Mathematics of NAS of Ukraine, Kyiv, Ukraine
[2]Taras Shevchenko National University of Kyiv, Kyiv, Ukraine
vitalij.bond@gmail.com, petravchuk@knu.ua

Recall that a classification problem in Linear Algebra is called *wild* if it contains the problem of classifying pairs of $(n \times n)$-matrices up to simultaneous similarity

$$(A, B) \longmapsto S^{-1}(A, B)S = (S^{-1}AS,\ S^{-1}BS)$$

for some invertible matrix S. Otherwise, when indecomposble objects are "parameterized" by several discrete and at most one continuous parameters, the problem is called *tame*. These concepts (including the terms themselves) were first introduced by P. Donovan and M. R. Freislich in [1]. Formal definitions of these two classes were proposed by Yu.A.Drozd in [2], [3]; in these papers he also proved his well-known theorems about tame and wild problems.

Let K be a field, $R = K[x, y]$ the polynomial ring and $\mathcal{M}(K)$ the set of all pairs of square matrices of the same size over K. Pairs

$$P_1 = (A_1, B_1),\ P_2 = (A_2, B_2)$$

from $\mathcal{M}(K)$ are called similar if $A_2 = X^{-1}A_1X$ and $B_2 = X^{-1}B_1X$ for some invertible matrix X over K. Denote by $\mathcal{N}(K)$ the subset of $\mathcal{M}(K)$, consisting of all pairs of commuting nilpotent matrices. A pair P will be called *polynomially equivalent* to a pair $\overline{P} = (\overline{A}, \overline{B})$ if

$$\overline{A} = f(A, B), \overline{B} = g(A, B)$$

for some polynomials $f, g \in K[x, y]$ satisfying the next conditions: $f(0, 0) = 0, g(0, 0) = 0$ and $\det J(f, g)(0, 0) \neq 0$, where $J(f, g)$ is the Jacobi matrix of polynomials $f(x, y)$ and $g(x, y)$. Further, pairs of matrices $P(A, B)$ and $\widetilde{P}(\widetilde{A}, \widetilde{B})$ from $\mathcal{N}(K)$ will be called *polynomially similar* if there exists a pair $\overline{P}(\overline{A}, \overline{B})$ from $\mathcal{N}(K)$ such that P, $\overline{P}$ are polynomially equivalent and $\overline{P}$, $\widetilde{P}$ are similar. The problem of classifying pairs of matrices up to polynomial similarity can be reduced to the problem of classifying pairs (A, B) of matrices up to polynomial similarity in the case when

$$A^2 = 0, B^3 = 0, AB^2 = 0. \qquad (*)$$

As the next theorem shows this problem is wild, i.e., it contains the classical unsolvable problem of classifying pairs of matrices up to similarity.

Theorem. *The problem of classifying, up to polynomial similarity, pairs of matrices satisfying the equations $(*)$ is wild.*

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Software Optimization Means Survey

V. A. Borodin[1]

[1]Taras Shevchenko National University of Kyiv, Kyiv, Ukraine

viktorborodin@knu.ua

Optimization problem formulation in the form:

$$\min \quad f(x) \quad \text{(Objective function)} \tag{1}$$

$$\text{s.t.} \quad g(x) \leqslant b \quad \text{(Constraints)} \tag{2}$$

is one of the most universal and popular applied math problems. Therefore, it is often necessary to solve this problem numerically using computer software (SW).

There is a plethora of available SW designed to solve optimization problems (Optimization SW). However, there are some general bafflements about how to select, use, and analyze the results of the applications that use it.

This talk gives a brief review of existing up-to-date Optimization SW. The following specific questions are covered: steps of working with Optimization SW, what they consist of, why there are so many of them, recent advances in this field, their issues, and how to choose appropriate SW for a specific task in the real-world situation.

At first, optimization problems taxonomy and how its versatile qualities lead to the creation of different SW by nature is revealed. The main components of optimization software, Modeling SW, and Solvers, are explained, with examples of different problem types and formats used for the model representation. The scheme of the work with them is shown. Modeling SW is explained as declarative programming instances with the main formats of the models presented. The table with the most popular modeling frameworks and recommendations for choosing the required SW is delivered to the audience.

Different kinds of existing Solvers are discussed in the talk. Explained their principles of operation, how they are tested/evaluated, and the major issues in creating such kinds of SW. Some recent updates to the existing Solvers are presented.

Based on the given analysis, basic recommendations on when, how, and which Optimization SW to use for a particular problem are given. Concrete examples of Python implementations for optimization problems using the packages discussed are displayed.

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On monotonicity of some entropies

V. B. Braiman[1]

[1]Taras Shevchenko National University of Kyiv, Kyiv, Ukraine

volodymyr.braiman@knu.ua

In [1], it was shown that the Shannon entropy and the Rényi entropy of the Poisson distribution with parameter $\lambda > 0$ are increasing as functions of λ. In [2], it was shown that the Tsallis entropy and the Sharma-Mittal entropy of the Poisson distribution are also increasing as functions of λ, while the generalized Rényi entropy may be non-monotonic.

In this report, we consider sufficient conditions on entropy and on a family of discrete probability distributions that ensure the monotonicity of entropy.

Theorem. *Let* $\mathcal{P} = \left\{(p_0, p_1, \ldots, p_n, \ldots) \in \ell_1 : p_i \geqslant 0, i \geqslant 0, \sum_{i\geqslant 0} p_i = 1\right\}$, $H : \mathcal{P} \to \mathbb{R}$ *be an entropy such that*

1) $H \in C(\mathcal{P})$,

2) *for every* $n \geqslant 1$ *the function* $H(p_0, \ldots, p_n, 0, \ldots)$ *is Schur-concave on the set*

$$\left\{(p_0, p_1, \ldots, p_n) \in \mathbb{R}^{n+1} : p_i \geqslant 0, 0 \leqslant i \leqslant n, \sum_{i=0}^{n} p_i \leqslant 1\right\}.$$

Let $\{p_i(\lambda), i \geqslant 0\}$ *be a discrete probability distribution with parameter* λ *such that*

3) *for every* $n \geqslant 0$ *the sum* $\sum_{i=0}^{n} p_{[i]}(\lambda)$ *is decreasing as a function of* λ, *where* $\{p_{[i]}(\lambda), i \geqslant 0\}$ *is the sequence of numbers* $\{p_i(\lambda), i \geqslant 0\}$ *which are rearranged in non-increasing order.*

Then $H(p_i(\lambda), i \geqslant 0)$ *increases as a function of* λ.

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Limit behavior of randomly disturbed sequences

Mykola Denysievskyi
Taras Shevchenko National University of Kyiv, Kyiv, Ukraine
mykola.denysya@knu.ua

Consider a discrete dynamical system in $\mathbb{R}^d$ with random disturbances $\{\xi_n : \; n \geqslant 0\}$

$$X_{n+1} = F(X_n, \, \xi_n), \; n \geqslant 0.$$

For given function $f : \mathbb{R}^d \to \mathbb{R}^d$ we are interested in the behavior of sums

$$\sum_{k=0}^{n} f(X_k) \to ?, \; n \to \infty.$$

The concepts and some important facts used here are presented in the article [1].

Theorem. *Let random variables $\{\xi_n\}$ and function $F : \mathbb{R}^{2d} \to \mathbb{R}^d$ satisfy conditions:*

1. *Random variables $\{\xi_n : \; n \geqslant 0\}$ are independent with absolutely continuous distributions with respect to the Lebesgue measure with densities $\{p_n : \; n \geqslant 0\}$ which possess the following properties*
   (a) *there are a Borel function $p : \mathbb{R}^d \to (0; +\infty)$ and a sequence of positive numbers $\{a_n : n \geqslant 0\}$, $a_n \to +\infty$, $n \to \infty$, such that*
   $$0 < \alpha_n := \inf \left\{ \frac{p_n(t)}{p(t)} | \; \|t\| \leqslant a_n \right\} \leqslant \sup \left\{ \frac{p_n(t)}{p(t)} | \; \|t\| \leqslant a_n \right\} =: \beta_n < +\infty;$$
   (б) *$\alpha_n \to 0, \; n \to \infty$; $\alpha_n \sim \beta_n, \; n \to \infty$;*
   (в) *$\forall m \geqslant 1, \; r = 0, 1, 2, \ldots, m-1$:*
   $$\sum_{n=0}^{\infty} \alpha_{nm+r} = +\infty.$$
2. *There exists a Borel function $\psi : \mathbb{R}^d \to \mathbb{R}^d$ such that*
   $$\exists L \; \forall t, \, s_1, \, s_2 \in \mathbb{R}^d : \; \|F(s_1, t) - F(s_2, t)\| \leqslant L \|\psi(s_1) - \psi(s_2)\|,$$
   $$\exists N \; \forall s_1, \, s_2 \in \mathbb{R}^d : \; \|\psi(s_1) - \psi(s_2)\| \leqslant N \|s_1 - s_2\|, \; L \cdot N < 1.$$
3. $\|F(0, \, t)\| \to +\infty, \; \|t\| \to +\infty.$
4. *There exists a Borel function $F_0 : \mathbb{R}^d \to \mathbb{R}^d$ such that*
   $$\forall t \in \mathbb{R}^d : \; F(s, \, t) \to F_0(t), \; \|s\| \to +\infty.$$

Then sequence $\{X_n : \; n \geqslant 0\}$ has w.p. 1 local sojourn measures of the form $\{\gamma_n \sigma : \; n \geqslant 0\}$ where

$$\gamma_n = \sum_{k=0}^{n} \alpha_k, \; n \geqslant 0; \quad \sigma(\Delta) = \int_{F_0^{-1}(\Delta)} p(t) \, dt$$

for bounded Borel set $\Delta \subset \mathbb{R}^d$.

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Evolution of correlations in a system of hard spheres

I. V. Gapyak
Taras Shevchenko National University of Kyiv, Kyiv, Ukraine
ihapiak@knu.ua

We discuss mathematical problems related to the description of the evolution of many hard spheres using various approaches to characterize their state, particularly through functions that describe the propagation of correlations. The proposed approach enables the analysis of systems with both finite and infinite average numbers of hard spheres by means of reduced distribution functions or reduced correlation functions, whose dynamics are determined by the evolution of correlations in a hard-sphere system [1]. Special attention is given to the processes of correlation creation and propagation [2], which are closely connected to the problem of describing memory effects in many-particle systems with collisional dynamics.

It was established that the notion of cumulants of the groups of operators underlies nonperturbative expansions of solutions for the fundamental evolution equations describing the state evolution of a hard-sphere system, namely, of the Liouville hierarchy for correlation functions, of the BBGKY hierarchy for reduced distribution functions and of the nonlinear BBGKY hierarchy for reduced correlation functions, as well as it underlies the kinetic description of infinitely many hard spheres. We emphasize that the structure of obtained expansions for correlation functions, in which the generating operators are the cumulants of the corresponding order of the groups of operators of hard spheres, induces the cumulant structure of series expansions for reduced distribution functions, reduced correlation functions and reduced correlation functionals. Thus, the dynamics of systems of infinitely many hard spheres is generated by the dynamics of correlations.

The origin of the collective behavior of a hard-sphere system on a microscopic scale was described by means of a one-particle correlation function that is determined by the non-Markovian Enskog kinetic equation. The advantages of such an approach to the derivation of kinetic equations from underlying collisional dynamics consists of an opportunity to construct the kinetic equations with initial correlations, which makes it possible to describe the propagation of initial correlations in the Boltzmann-Grad limit [3].

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Vandermonde operator and linear differential equations of higher orders

M. F. Horodnii

Taras Shevchenko National University of Kyiv, Kyiv, Ukraine

horodnii@knu.ua

We study the question of the existence of a unique bounded solution, as well as of the Cauchy problem to a higher-order linear differential equation with bounded operator coefficients. The case is considered when the corresponding "algebraic" operator equation has separated pairwise commuting roots. Using the Vandermonde operator constructed from such roots, representations of a unique bounded solution and the Cauchy problem are obtained.

Correlation-Regression Analysis of Consumer Market Behavior and Its Impact on Investment Patterns in the Eurozone

M. V. Hrysenko[1]

[1]Taras Shevchenko National University of Kyiv, Kyiv, Ukraine

magryss@knu.ua

The aim of the study is to examine the relationship between investor sentiment and Eurozone exchange-traded funds (ETFs) in order to understand the dynamics of market behavior and the impact of consumer behavior on investment patterns in the Eurozone. Panel data analysis, correlation analysis, regression analysis and Granger causality test are used to investigate the relationship between ETFs and the Consumer Confidence Index (CCI). The results show that the CCI has a significant negative impact on ETF pricing. Other factors such as financial stability, government regulation and market factors can also affect ETF prices. Therefore, it can be concluded that while the CCI may be important, it should not be the only factor considered.

This study used several data analysis methods to investigate the relationship between ETFs and CCI in the Eurozone. The first method is panel data analysis. Panel data regression models are useful for examining correlations between variables in situations where the data contain both cross-sectional and time series components. By taking into account the temporal dynamics within each market, this method allows researchers to account for potential volatility across markets. The panel data model includes 7 developed markets in the Eurozone, such as Austria, Belgium, France, Germany, Italy, the Netherlands, and Spain. The analysis of this study covers the period from March 1, 2015 to March 1, 2023. After conducting a panel data analysis, this study decided to look at the hypothesis from a different perspective and investigate the relationship between the iShares MSCI Eurozone ETF and the CCI EA19 using additional methods such as correlation analysis, regression analysis, and Granger causality testing. Correlation analysis was used to examine the linear relationship and measure the strength and direction of the relationship between the two variables. This type of analysis is useful for understanding the extent to which ETFs and CCI are related and for identifying any potential patterns in their relationship. Regression analysis was conducted to assess the impact of CCI on the iShares MSCI Eurozone ETF.Regression analysis helps to better understand the extent to which CCI can affect the iShares MSCI Eurozone ETF. The combination of methods considered helps to explore the nature of the relationship between consumer confidence and ETF pricing in Eurozone markets in a comprehensive and robust manner.

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Solvability Issue for Optimal Control Problem for Degenerate Parabolic Variational Inequality

N. V. Kasimova[1]

[1]Taras Shevchenko National University of Kyiv, Kyiv, Ukraine

kasimova@knu.ua

We investigate the problem of solvability for optimal control problem for degenerate parabolic variational inequality in the so called "weak" statement.

Let $\Omega \in \mathbb{R}^N$ be a bounded open subset with rather regular boundary $\partial\Omega$ and let $0 \in \mathbb{R}^N$ be the inner point of the set Ω. Let $Q = (0,T) \times \Omega$ be a cylinder in $\mathbb{R}^1 \times \mathbb{R}^N$, where $T < +\infty$. We denote its side surface by $\Sigma = (0,T) \times \partial\Omega$, and let the boundary of Ω is divided into two subsets with positive measure $\partial\Omega = \Gamma_D \cup \Gamma_N$.

We consider the following problem

$$I(u,y) = \frac{1}{2}\left\| y - \frac{y_{ad}}{\sqrt{\rho}} \right\|^2_{L^2(0,T;L^2(\Omega,\rho dx))} + \frac{1}{2}\|u\|^2_{L^2(0,T;L^2(\Gamma_N;\rho^{-1}d\xi))} \to \inf, \tag{1}$$

$$\int_0^T\int_\Omega \dot{v}\cdot(v-y)\rho dxdt + \int_0^T\int_\Omega (\nabla y, \nabla(v-y))_{\mathbb{R}^N}\rho dxdt \geqslant$$

$$\geqslant \int_0^T\int_\Omega f\cdot(v-y)dxdt + \int_0^T\int_{\Gamma_N} u\cdot(v-y)d\xi dt \tag{2}$$

$$\forall v \in \mathcal{K},\ \dot{v} \in L^2(0,T;(W^{1,2}(\Omega;\Gamma_D,\rho dx))^*), v(0,x) = 0,$$

$$u \in U_\partial,\ y \in \mathcal{K}, \tag{3}$$

$$y(0,x) = 0,\ \ x \in \Omega. \tag{4}$$

Here y_{ad} and f are given distributions, $W^{1,2}(\Omega;\Gamma_D,\rho dx)$ is the corresponding weighted Sobolev space, $\mathcal{K}$ is a convex closed subset of the space $L^2(0,T;W^{1,2}(\Omega;\Gamma_D,\rho dx))$, U_∂ is a nonempty convex closed subset of $L^2(0,T;L^2(\Gamma_N;\rho^{-1}d\xi))$.

Such problems without degenerations are well-known and we have the classical results concerning their solvability. These results require such standard conditions as coercivity and boundedness for an elliptic operator connected to the problem. In considered problem the weighted function ρ could be unbounded or attain the value zero on the boundary layer of the domain. Therefore, we can't use the classical theorems. And our aim is an obtaining of sufficient conditions for function ρ, under which the given optimal control problem has a solution. For this purpose, we consider a transformation, according to which the original problem is reduced to the optimal control problem for a parabolic variation inequality with unbounded coefficients of the potential type, and the issue of the existence of its unique solution is investigated using the Hardy-Poincar′e type inequality. Thus, the main result of our investigation is the next theorem [1].

Theorem. *Let $\rho : \Omega \to \mathbb{R}_+$ be a weight function of potential type. Let f, y_{ad} are given functions. Then OCP (1)-(4) admits a unique solution (u^0, y^0) in the space $L^2(0,T;L^2(\Gamma_N,\rho^{-1}d\xi)) \times L^2(0,T;W^{1,2}(\Omega;\Gamma_D,\rho dx))$.*

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Asymptotic approximations for the solution of a singularly perturbed convection-diffusion problem in a thin graph-like junction

A. V. Klevtsovskiy[1]

[1]Taras Shevchenko National University of Kyiv, Kyiv, Ukraine

avklevtsovskiy@knu.ua

Convection-diffusion problems have recently attracted a lot of attention in the research literature (see e.g. monograph [1] and references there). Accurate modeling of the interaction between convective and diffusion processes is the most common and difficult problem in the numerical approximation of partial differential equations. Justification both numerical approximations and asymptotic approximations for solutions to convection-dominated problems is more hard task. The reason is that the corresponding solutions have boundary layers, i.e., there are narrow areas where solutions are bounded regardless of small parameter near diffusion term, but where their derivatives are large [1, Section 1.1].

It our talk we present the construction of the complete asymptotic expansion for the solution to a steady-state convection-dominated problem in a thin three-dimensional graph-like junction consisting of three thin cylinders connected through a domain (node) of diameter $\mathcal{O}(\varepsilon)$. In addition, we consider the Dirichlet boundary conditions on the bases of the thin cylinders and the perturbed nonhomogeneous Neumann type boundary condition on the lateral surfaces, in which the flows of both the convective and diffusion parts are involved.

Using multiscale analysis, the asymptotic expansion for the solution is constructed and justified. The asymptotic estimates in the norm of Sobolev space H^1 as well as in the uniform norm are proved for the difference between the solution and proposed approximations with a predetermined accuracy with respect to the degree of ε.

These results were obtained in collaboration with Prof. Melnyk in the work [2].

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Sobolev inequality from the stochastic perspective

V. Knopova[1]

[1]Taras Shevchenko National University of Kyiv, Kyiv, Ukraine

vicknopova@knu.ua

My talk is devoted to the relation between Poincaré, (log)Sobolev and concentration inequalities. We discuss “stochastic” versions of Poincaré and (log)Sobolev inequalities, the Herbst argument for deriving the concentration inequality, as well as some examples.

Mixing of Viscous Fluids in a Rectangular Cavity by Ghost Rods

O. B. Kurylko

Taras Shevchenko National University of Kyiv, Kyiv, Ukraine

alexandr.kurylko@knu.ua

Abstract: This study investigates the mixing of viscous fluids in a rectangular cavity using a novel approach involving ghost rods. The research is inspired by the topological fluid mechanics framework introduced by Boyland, Aref, and Stremler [1]. We analyze fluid motion under various mixing protocols, particularly pseudo-Anosov and finite-type protocols, which are shown to differ in efficiency despite being energetically equivalent.

Problem statement: The steady-state Stokes flow is considered in a rectangular domain with tangential boundary velocities prescribed in a piecewise-constant fashion. The biharmonic stream function formulation is applied, and the solution is constructed using symmetry arguments involving even and odd components [2].

Key contributions:

- Development and analysis of several mixing protocols using alternating boundary velocities.
- Numerical visualization of streamlines and Poincare sections, revealing the presence of periodic points and Lagrangian structures.
- Introduction of the concept of "ghost rods" to interpret flow behavior and mixing efficiency.
- Comparison of advection patterns under different protocols (e.g., R^+L^+, R^+L^-) using tracer particles and streamline plots.

Conclusion: The method of ghost rods provides an intuitive and mathematically rigorous way to control and analyze viscous mixing in constrained geometries. Protocols of the pseudo-Anosov type lead to more effective mixing, as demonstrated through computational experiments. The concept of ghost rods introduces a flexible framework for designing efficient mixers without physical obstacles, which is particularly useful in microfluidics and lab-on-a-chip systems where non-invasive control is essential. This theoretical framework may be extended to three-dimensional flows and time-dependent boundary conditions.

Keywords: Viscous fluid, Stokes flow, mixing protocol, ghost rod, pseudo-Anosov, rectangular cavity, Poincare section.

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On the Instability and Some Asymptotic Properties of Random Processes Associated with the Bessel Diffusion Process

S. V. Kushnirenko[1]

[1]Taras Shevchenko National University of Kyiv, Kyiv, Ukraine

svilana_kushnirenko@knu.ua

Let $(\Omega, \mathcal{F}, \mathbf{P})$ be a probability space and $W = \{W_t, t \geqslant 0\}$ be a Wiener process on it. We study stochastic differential equation, namely

$$Y_t = y_0 + at + \sigma \int_0^t \sqrt{Y_s}\, dW_s, \tag{1}$$

where $y_0 > 0$, $a > 0$, and $\sigma > 0$.

It is well known that equation (1) admit unique strong solution $Y = \{Y_t, t \geqslant 0\}$. The process Y is commonly referred as the squared Bessel process. Strictly positive values of the process Y make it convenient to model real processes in physics, biology, economics, and finances.

We investigate this process using the concept of stochastic instability. We demonstrate that the squared Bessel process Y is stochastically instable, i.e., for any constant $N > 0$

$$\lim_{t \to +\infty} \frac{1}{t} \int_0^t \mathbf{P}\{Y_s < N\} ds = 0.$$

For a smooth version of Bessel process, i.e., a solution of the stochastic differential equation

$$dV_t^\varepsilon = \frac{cdt}{\sqrt{(V_t^\varepsilon)^2 + \varepsilon^2}} + dW_t,$$

where $\varepsilon \neq 0$, $c > 0$, $V_0^\varepsilon = \sqrt{y}_0 > 0$, we establish the convergence of V^ε to the (non-squared) Bessel process $V = \sqrt{Y}$, as $\varepsilon \to 0$.

We also demonstrate stochastic instability of V^ε. And finally, we establish a bit unexpected statement for fixed $\varepsilon \neq 0$.

Theorem. (Theorem VI.7 from [1]) *Normalised stochastic process $\frac{V_{tT}^\varepsilon}{\sqrt{T}}$ converges weakly, as $T \to \infty$, to the Bessel process Y_t that is the solution of the equation*

$$Y_t^2 = 3t + 2\int_0^t Y_s\, dW_s, \quad t \geqslant 0.$$

Proof of the Theorem is based on Theorem 3.3 from book [2].

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Application of physics informed neural networks to solving problems of the non-homogeneous elasticity

M. V. Lavrenyuk[1]

[1]Taras Shevchenko National University of Kyiv, Kyiv, Ukraine

m.lavrenyuk@knu.ua

A general approach for simulating the behavior of geological systems by calculating the stress-strain state of non-homogeneous regions subjected to gravitational forces has been developed. This approach reduces boundary value problems in solid mechanics to unconstrained optimization problems. Using Physics-Informed Neural Networks (PINNs) within this framework simplifies the solution to constructing substitution functions that depend on the boundary and initial conditions and the neural network solution of the corresponding optimization problem. The problem of non-homogeneous plane elasticity was analyzed within this approach. For Neumann conditions set on the top and right sides, and Dirichlet conditions on the left and bottom sides of a trapezoid-shaped region with inhomogeneous mechanical properties, the ansatz functions for displacements for the unconstrained optimisation problem were obtained. Constructed functions satisfy the boundary conditions of the problem by construction. and the order of the coordinate functions in only 5.

To verify the proposed methodology, a series of checks to ensure the accuracy of our approach were performed. The first problem considered was the stress-strain state of a homogeneous half-plane subjected solely to gravitational forces. The resulting stresses were consistent with the known classical results. The second problem involved the same half-plane under gravitational force, but considered as a non-homogeneous region. The special case of non-homogeneity was examined. In this case stresses were the same as for the homogeneous case of the first problem and were consistent with the analytical solution. The third problem involved calculating the stress-strain state for a two-dimensional non-homogeneous elasticity problem of a rectangular region under gravitational forces. The region was fixed at the left side and the bottom, while the right side and the top were free. Numerical results for this case were not included in this study, instead the more general fourth problem of defining the stress-strain state of a two-dimensional non-homogeneous elastic trapezoid-shaped region under gravitational forces was considered. The proposed methodology can be extended to problems in the theory of thermal elasticity, including piecewise-homogeneous and thermosensitive media.

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Mobile Application for Cognitive and Memory Training in a Game-Based Format

A. I. Lisnyi
Taras Shevchenko National University of Kyiv, Kyiv, Ukraine
sefelan@knu.ua

This paper presents the design and implementation of **Grydly**, an Android application aimed at improving users' cognitive abilities through interactive games. The project follows the Clean Architecture approach [1], separating the app into three layers: Presentation, Domain, and Data. Each game (e.g., Fifteen Puzzle, Lights Out) is implemented with isolated logic, progress tracking, and reusable domain components.

The application was built using Kotlin, MVVM architecture, Room for local storage, and Hilt for dependency injection [2]. Game logic is encapsulated in use cases, making it scalable and testable. The app supports both Campaign and Quick Play modes and includes visual performance metrics.

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On perturbation of invariant tori that foliate the central manifold of conditionally integrable locally Hamiltonian systems

Yu.V. Loveikin
Taras Shevchenko National University of Kyiv, Kyiv, Ukraine
y.loveikin@knu.ua

In the class of locally Hamiltonian systems, the problem of perturbation of a conditionally integrable system is studied. Conditionally integrability of a system means that its invariant tori do not foliate an open region of the phase space, but only some submanifold $\mathcal{M}_0$ (the central manifold). The main goal is to show: if the corresponding linearized system in the neighborhood of the specified submanifold $\mathcal{M}_0$ is in a certain sense hyperbolic in the normal direction to $\mathcal{M}_0$, then the system obtained as a result of small locally Hamiltonian perturbations of the initial conditionally integrable system has an invariant manifold (central manifold) $\mathcal{M}$ which is a small deformation of the $\mathcal{M}_0$ and on it there exists a Cantor subset of invariant tori of the perturbed system carring quasiperiodic motions.

On a smooth symplectic manifold $(M^{2(n+k)}, \omega^2)$ with symplectic structure ω^2, we consider a locally Hamiltonian system with a closed (not necessarily exact) 1-form ω_0^1 (instead of the 1-form ω_0^1, we can consider a multivalued Hamiltonian function $H_0 : M^{2(n+k)} \to \mathbb{R}$). We assume that this system is conditionally integrable. Namely, this system has a $2n$-dimensional invariant submanifold $\mathcal{M}_0$, which is foliated by its invariant tori carring quasiperiodic motions.

Let this system suffers a perturbation of the form

$$\omega_0^1 \mapsto \omega_0^1 + \mu\omega_1^1 \quad \text{(or the same } H_0 \mapsto H_0 + \mu H_1\text{)},$$

where $\mu > 0$ — small parameter, $H_0 + \mu H_1$ — multivalued Hamiltonian function corresponding to 1-form $\omega_0^1 + \mu\omega_1^1$.

It was established that certain conditions for an unperturbed conditionally integrable system under the small perturbations the resulting perturbed system has an invariant (central) manifold $\mathcal{M}$ which is a small deformation of the manifold $\mathcal{M}_0$. On this invariant manifold $\mathcal{M}$ the system induced by the initial perturbed system is also locally Hamiltonian and this induced system has invariant tori that carry quasiperiodic motions. The obtained results were established with the use of the methods of the KAM theory using the smoothing method proposed by J. Moser in combination with the method of artificial parameters.

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Multifractal Properties of Human Heart Rhythm

V. T. MATSYPURA
Taras Shevchenko National University of Kyiv, Kyiv, Ukraine
mnivtt@knu.ua

The human heart is the source of the main biological signal - the heart rate. Recording of skin potentials induced by the heart muscle (electrocardiogram (ECG)) provides information about the electrical activity of the atria and ventricles. Since the electrical activity of the human heart is subject to the influence of many physiological mechanisms, electrocardiography has become an indispensable tool for diagnosing pathologies of the cardiovascular system.

For a long time, it was believed that the heart rhythm should be constant in order to ensure the stable functioning of the whole organism. That is why it was believed that when a person is sick or gets old, it is more difficult for his body to maintain a constant heart rhythm and the degree of its variation increases.

However, modern research has overturned the long-standing ideas of doctors. Analysis of heart rate has shown that its variability (high degree of variability) is inherent in young and healthy organisms, because it is precisely such chaotic dynamics that allow our body to adapt to sudden changes in the environment, while a decrease in heart rate variation, on the contrary, is inherent in older people or people with certain health changes. Analysis of changes in the duration of the sequence of intervals between heartbeats turned out to be very important from the point of view of assessing the presence or possibility of cardiovascular diseases.

Perhaps the human intervalogram was among the first biological signals that were studied from the point of view of chaos theory and fractal analysis. Later, these studies were significantly expanded and received a large bibliography.

The report presents a comparative analysis of multifractal spectra of a healthy person and a person with a certain heart disease. Its results give hope that multifractal analysis of experimental data of signals of medical and biological origin will reveal new information characteristics, which, on the one hand, will help to understand the nature of the human body more deeply, and, on the other, will become information parameters in the diagnosis of human health.

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Structure recovery: from mean estimation to higher moments

Shahar Mendelson[1]

[1]Institute for Mathematical Research, ETH Zurich, Switzerland

shahar.mendelson@gmail.com

Let X be a random vector in R^d with a finite mean and covariance. I focus on two questions:

- First, if the mean and covariance of X are not known, and instead one is given $X_1, ..., X_N$ that are independent copies of X, is it possible to "guess" the mean EX - relative to a fixed norm on R^d, with high accuracy and confidence?
- And second, if $T \subset S^{d-1}$ and given $X_1, ..., X_N$, is it possible to accurately guess

  $$\| < X, t > \|_{L_p(X)}$$

  uniformly in $t \in T$ - (assuming some partial information on the mean and covariance of X)?

The first question is natural, and I will explain that the second is connected to a question asked by V. Milman on the behaviour of isotropic, log-concave random vectors. I will present an answer to the first question and then show that (optimal) mean estimation in R together with Talagrand's generic chaining mechanism leads to a sharp answer to the second question.

On modules of continuity of fractional orders which are generated by a semigroup of operators

O. N. Nesterenko
Taras Shevchenko National University of Kyiv, Kyiv, Ukraine
nesterenkoon@ukr.net

The report was prepared based on the results obtained jointly with S. I. Bezkryla and A. V. Chaikovskyi and published in the works [1], [2].

Let X be a linear space, $\{T_h \ : \ h \geqslant 0\}$ - a one-parameter family of linear operators $T_h : X \to X, \ h \geqslant 0$, which forms a semigroup, i.e. $T_0 = I$ - a unit operator and $T_{h_1+h_2} = T_{h_1}T_{h_2}$ for arbitrary $h_1 \geqslant 0, \ h_2 \geqslant 0$. We assume that there exists a linear set $Y \subset X$, on which the norm $\| \cdot \|$ is introduced, relative to which the space Y is Banach space, and for all $f \in X$ and $h \geqslant 0$ there is an inclusion $(T_h - I)f \in Y$ and $\|T_h f - f\| \to 0, \ h \to 0+$. Then for all $h \geqslant 0$ the operator $T_h : Y \to Y$. Let also for each $h \geqslant 0$ restriction of the operator T_h to Y, which we will denoted $\tilde{T}_h$, be a continuous operator and its norm $\|\tilde{T}_h\| \leqslant 1$.

Next, we distinguish the cases $X = Y$ and $X \neq Y$. If $X = Y, \ f \in X, \ \alpha > 0$, then $(I - T_h)^\alpha f := \sum\limits_{j=0}^{\infty} C_\alpha^j (-1)^j T_h^j f$. If $X \neq Y, \ f \in X, \ \alpha > 1, \ g = (I - T_h)f$ then $(I - T_h)^\alpha f := (I - T_h)^{\alpha-1} g$ where $(I - T_h)^{\alpha-1} g := \sum\limits_{j=0}^{\infty} C_{\alpha-1}^j (-1)^j T_h^j f$.

Definition. Let $\alpha > 0$ if $X = Y$ and $\alpha \geqslant 1$ if $X \neq Y$. The modulus of continuity of an element $f \in X$ of order α, generated by the semigroup $\{T_h \ : \ h \geqslant 0\}$, is the function

$$\omega_\alpha(t) := \omega_\alpha(f, t) := \sup_{h \in [0,t]} \|(I - T_h)^\alpha f\|, \ t \geqslant 0.$$

Lemma. Let $\alpha > 0$ if $X = Y$ and $\alpha \geqslant 1$ if $X \neq Y$. Let also $\omega(\cdot) := \omega_\alpha(f, \cdot)$ be the modulus of continuity of an element $f \in X$ of order α, generated by the semigroup $\{T_h \ : \ h \geqslant 0\}$. Then the following properties hold:

1) $\omega(0) = 0$; 2) $\omega \uparrow$ on $[0, +\infty)$; 3) $\omega \in C([0, +\infty))$;

4) if $\alpha = k \in \mathbf{N}$, then $\omega(nt) \leqslant n^k \omega(t)$ for all $t \geqslant 0$ and $n \in \mathbf{N}$.

Theorem 1 [2]. *Let $X = Y$ and $\alpha \geqslant 3$ or $X \neq Y$ and $\alpha \geqslant 4$. Let also $\omega(\cdot) := \omega_\alpha(f, \cdot)$ be the modulus of continuity of an element $f \in X$ of order α, generated by the semigroup $\{T_h \ : \ h \geqslant 0\}$. Then*

$$2\omega_\alpha(f, nt) \leqslant \omega_\alpha(f, (n+1)t) + \omega_\alpha(f, (n-1)t) + C_n \omega_\alpha(f, t), \ t > 0,$$

where C_n is a constant that depends only on α and n , and $C_n = O(n^{\alpha - 3/2}), \ n \to \infty$.

Theorem 2 [1]. *Let $X = Y$ and $\alpha \geqslant 3$ or $X \neq Y$ and $\alpha \geqslant 4$. Then for an arbitrary number $\beta > \alpha - \frac{1}{2}$ there exists a function $\omega : [0, +\infty) \to \boldsymbol{R}$, not identically equal to zero, which satisfy conditions 1) 3) of the lemma, such that the function $\frac{\omega(t)}{t^\beta}, \ t \in (0, +\infty)$, is monotonically non-increasing on $(0, +\infty)$ and at the same time the equality $\lim\limits_{t \to 0+} \frac{\omega_\alpha(f,t)}{\omega(t)} = 1$ is not satisfied for any element $f \in X$.*

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Stochastic center of systems of stochastic differential equations on the plane

O .V.Perehuda [1]

[1]Taras Shevchenko National University of Kyiv, Kyiv, Ukraine

perehudao@knu.ua

Let (Ω, F, P) be a complete probability space and $w(t)$ be a standard one-dimensional Wiener process on Ω.

Consider the system $$\begin{cases} dx(t) = f_1(x,y)dt + g_1(x,y)dw(t), \\ dy(t) = f_2(x,y)dt + g_2(x,y)dw(t), \end{cases} \quad (1)$$

where $f_1(x,y), f_2(x,y), g_1(x,y), g_2(x,y)$ -non-random real polynomials of the second degree that do not contain constants.

In the present work, we shall give a stochastic version of the center problem for systems of stochastic differential equations on the plane.

Definition. We say that system (1) has a stochastic center at the origin if there is a non constant analytic function $H(x,y)$ in a neighborhood U of the origin such that for any $(x_0, y_0) \in U$, if $(x(t), y(t))$ is the maximal solution of system (1) with the initial conditions $x(0) = x_0$, $y(0) = y_0$, then for all $0 \leqslant t < \sigma_(x_0, y_0)$

$$H(x(t), y(t)) - H(x_0, y_0) = 0 \quad a.s.$$

Observe that if the curve $\{(x,y) \in R^2 : H(x,y) = H(x_0,y_0)\}$ is closed, then $\sigma(x_0,y_0) = +\infty$.

Theorem. *The system (1) has a stochastic center, if and only if for all $(x,y) \in U$,*

$$g_1(x,y)\frac{\partial H}{\partial x}(x,y) + g_2(x,y)\frac{\partial H}{\partial y}(x,y) = 0, \quad (2)$$

$$f_1(x,y)\frac{\partial H}{\partial x} + f_2(x,y)\frac{\partial H}{\partial y} + \frac{1}{2}(g_1^2\frac{\partial^2 H}{\partial x^2} + 2g_1g_2\frac{\partial^2 H}{\partial x \partial y} + g_2^2\frac{\partial^2 H}{\partial y^2}) = 0 \quad (3)$$

Topological structure of simple Hamiltonian flows on the non-orientable surfaces

O. Prishlyak

Taras Shevchenko National University of Kyiv, Kyiv, Ukraine

prishlyak@knu.ua

We consider flows whose lift to the double cover are Hamiltonian flows with a Hamiltonian that is a Morse function. A flow is simple if there are no saddle connections between different saddles within it. Topological equivalence of flows is a homeomorphism of the surface that maps trajectories to trajectories and preserve their direction. For the classification of such flows, we use distinguishing graphs composed of Reeb graphs with distinguished leaves. The vertexes of the graph are coloured as A_0, A_1, A_2, B or C. Also, we mark one of incident half-edges for each vertex with degree more then one. An equivalence of graphs is an isomorphism that preserve colour of vertexes and marked half-edges.

Theorem. *Two simple Hamiltonian flows on non-oriented closed surfaces are topologically equivalent if and only if their distinguishing graphs are equivalent.*

An analytical expression for the number of topologically non-equivalent flows has been obtained for the projective plane and the Klein bottle.

Theorem. *The number of topologically non-equivalent simple Hamiltonian flows with k saddles on the projective plane* $\mathbb{R}P^2$ *can be calculated using the formula*

$$N(\mathbb{R}P^2)_k = K_k + \sum_{i=0}^{k-1} K_i K_{k-i-1},$$

where

$$K_{2n} = 3(K_0 K_{2n-1} + K_1 K_{2n-2} + \ldots + K_{n-1} K_n),$$

$$K_{2n+1} = 3(K_0 K_{2n} + K_1 K_{2n-1} + \ldots + K_{n-1} K_{n+1}) + \frac{3K_n^2 + K_n}{2}.$$

Theorem. *The Reeb graph on the Klein bottle has one of the following types:*

a) *the graph is a tree containing two vertices of degree 2, and the vertices of degree 1 are of type C;*

b) *the graph is a tree containing one vertex of degree 2, one vertex of type B, and the remaining vertices of degree 1 are of type C;*

c) *the graph is a tree that does not contain vertices of degree 2, has two vertices of type B, and the remaining vertices of degree 1 are of type C;*

d) *the graph is a tree that does not contain vertices of degree 2, has one vertex of type* A_2*, and the remaining vertices of degree 1 are of type C;*

e) *the graph has one simple cycle, this cycle contains an odd number of R-vertices, and all vertices of the graph are either of type C or of degree three.*

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General Stochastic Measures. Integration, Path Properties and Equations

V. M. Radchenko

Taras Shevchenko National University of Kyiv, Kyiv, Ukraine

vadymradchenko@knu.ua

Let $\mathsf{L}_0 = \mathsf{L}_0(\Omega,\ \mathcal{F},\ \mathsf{P})$ be a set of all real-valued random variables defined on the probability space $(\Omega,\ \mathcal{F},\ \mathsf{P})$. Convergence in L_0 means the convergence in probability. Let X be an arbitrary set and $\mathcal{B}$ be a σ-algebra of subsets of X.

Definition. *The σ-additive mapping $\mu:\ \mathcal{B} \to \mathsf{L}_0$ is called a stochastic measure (SM).*

The review of results concerning the properties of SMs and integrals with respect to SMs is given in the talk. The detailed theory of SMs may be found in [3].

Further, let SM μ defined on the Borel σ-algebra of $\mathbb{R}$.

Theorem. ([3]) *Let the process $\mu(t) = \mu([0,t])$, $0 \leqslant t \leqslant 1$, have continuous paths. Then for any $1 \leqslant p < +\infty$, $0 < \alpha < \min\{1/p, 1/2\}$ the trajectory $\mu(t)$ with probability 1 belongs to the Besov space $B^{\alpha}_{p,p}([0,1])$.*

Also, it was proved that, under some assumptions, the process $\mu(t)$, $0 \leqslant t \leqslant T$, admits a Fourier series expansion, see [3].

We consider the stochastic equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + f(t,x,u(t,x)) + \frac{\partial g}{\partial x}(t,x,u(t,x)) + \sigma(t,x)\frac{\partial \mu}{\partial x}, \quad u(0,x) = u_0(x) \tag{1}$$

where $(t,x) \in [0,T] \times \mathbb{R}$. If $f = 0$ and $g(x,t,v) = v^2/2$, then (1) is the Burgers equation, for $g = \mathrm{const}$, we get the heat equation.

Under some assumptions on elements of (1), it was proved that this equation has a unique mild solution which is continuous with values in $\mathsf{L}^2(\mathbb{R})$, see [4]. The averaging principle for (1) was proved.

The parabolic equation driven by SM, defined on sets of space variables, was considered in [1]. The heat equation on a bounded domain in $\mathbb{R}^n$ with SM, defined on sets of time variable, was studied in [2].

Also, the symmetric integral of the form $\int_{(0,T]} f(\mu_t, t) \circ d\mu_t$, $f \in \mathbb{C}^{1,1}(\mathbb{R}^2)$, was defined. We prove the statement about the existence and uniqueness of a solution to the equation

$$\circ\, dX_t = \sigma(X_t) \circ d\mu_t + b(X_t,t)\, dt, \quad 0 \leqslant t \leqslant T.$$

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Asymtotics of the number of automata over a binary alphabet that generate (in)finite groups

A. V. Russyev[1]

[1]Taras Shevchenko National University of Kyiv, Kyiv, Ukraine

andriyrussyev@knu.ua

Let X be a finite non-empty set. Denote by X^* the set of all words over the alphabet X including the empty word Λ. The length of word $w \in X^*$ is denoted by $|w|$.

A finite invertible automaton A over the alphabet X is a tuple $A = (X, Q, \varphi, \lambda)$, where Q is a finite set of states, $\varphi : Q \times X \to Q$ is a transition map, $\lambda : Q \times X \to X$ is an output map, and for each state $q \in Q$, the map $\pi_q : X \to X$ given by $\pi_q(x) = \lambda(q, x)$ is a permutation. The automaton A is called degenerate if there exists a permutation ρ such that $\pi_q = \rho$ for all states $q \in Q$.

The transition and output maps of the automaton A can be extended to the set $Q \times X^*$. For $q \in Q$, $w \in X^*$ and $x \in X$ we set

$$\varphi(q, wx) = \varphi(\varphi(q, w), x), \qquad \varphi(q, \Lambda) = q,$$
$$\lambda(q, wx) = \lambda(q, w)\lambda(\varphi(q, w), x), \qquad \lambda(q, \Lambda) = \Lambda.$$

The extension of the output map defines a map $f_q : X^* \to X^*$ for every state $q \in Q$ given by $f_q(w) = \lambda(q, w)$. The group generated by the set $\{f_q : q \in Q\}$ is denoted by $G(A)$ and called the automaton group generated by A.

The nucleus of the automaton A is a subset of the set Q defined by

$$\mathcal{N}(A) = \bigcap_{n \geqslant 0} \{\varphi(q, w) : q \in Q, w \in X^*, |w| \geqslant n\}.$$

Theorem 1 ([1]). *Let $A = (Q, X, \varphi, \lambda)$ be an invertible automaton over a binary alphabet X such that $\mathcal{N}(A) = A$. Suppose that there exist states $q_1, q_2 \in Q$ and letters $x_1, x_2 \in X$ such that $\varphi(q_1, x_1) = \varphi(q_2, x_2)$ and $\pi_{q_1} \neq \pi_{q_2}$. Then $G(A)$ is infinite.*

Let $\mathcal{A}_n$ be the set of invertible automata with n states and non-degenerate nucleus over the binary alphabet $\{0, 1\}$ and let $\mathcal{F}_n$ be its subset of automata A such that $\mathcal{N}(A)$ do not satisfy the conditions of Theorem 1.

Theorem 2. $\lim_{n\to\infty} |\mathcal{F}_n|/|\mathcal{A}_n| = 0$.

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Pseudodifferential equations with degeneration for radial finctions of p-adic argument

M. V. Serdiuk[1]

[1]Taras Shevchenko National University of Kyiv, Kyiv, Ukraine

mariiaserdiuk@knu.ua

Let $\alpha > 0$, $\gamma > 0$. We consider the problem

$$|t|_p^\gamma (D^\alpha u)(|t|_p) = f(|t|_p, u(|t|_p)),\ 0 \neq t \in \mathbb{Q}_p, \tag{1}$$

$$u(0) = u_0, \tag{2}$$

where

$$(D^\alpha \varphi)(t) = \frac{1-p^\alpha}{1-p^{-\alpha-1}} \int\limits_{\mathbb{Q}_p} |y|_p^{-\alpha-1} \left[\varphi(t-y) - \varphi(t)\right] dy$$

is the Vladimirov-Taibleson operator of fractional differentiation on a non-Archimedean local field $\mathbb{Q}_p$.

With the problem (1)–(2) we associate the integral equation

$$u(|t|_p) = u_0 + I^\alpha \left[|\cdot|_p^{-\gamma} f(|\cdot|_p, u(|\cdot|_p))\right](|t|_p), \tag{3}$$

where the fractional integral I^α is the right inverse operator of D^α.

We call a solution u to the equation (3), if it exists, a mild solution to the Cauchy problem (1)–(2).

Theorem. *Suppose that $\gamma < \min(1, \alpha)$ and the function $f : p^{\mathbb{Z}} \times \mathbb{R} \to \mathbb{R}$ satisfies the conditions*

$$|f(|t|_p, x)| \leqslant M,$$

$$|f(|t|_p, x) - f(|t|_p, y)| \leqslant F|x-y|,$$

for all $t \in \mathbb{Q}_p$, $x, y \in \mathbb{R}$ and some constants M, F independent on t, x, y. Then the problem (1)–(2) has a unique local mild solution.

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Noetherian boundary value problem for a system of dynamic equations on a time scale

T. V. Shovkoplias[1]

[1]Taras Shevchenko National University of Kyiv, Kyiv, Ukraine

1zagmat.tetyana1@knu.ua

A linear Noetherian boundary value problem for a system of second-order dynamic equations is considered on a time scale

$$(P(t)x^{\triangle}(t))^{\triangle} - Q(t)x(t) = f(x), t \in [a,b]_T. \tag{1}$$

$$lx(\cdot) = \alpha, \alpha \in R^m. \tag{2}$$

This boundary value problem is considered in the case when the operator of the linear part is irreversible, that is the number of boundary conditions of the problem and the dimensionality of the system of dynamic equations are different (such boundary value problems are called Noetherian).

In order to establish the solvability conditions of the boundary value problem under consideration, the apparatus of the theory of pseudoinverse matrices is used.

A connection is established between the solvability condition of the dynamical system and the solvability condition of the algebraic system of equations.

That is using the theory of pseudoinverse matrices, a condition for the solvability of dynamic systems of equations has been established to which the considered boundary value problem is redused.

In this case the solvability condition of the dynamic system of equations follows from the solvability condition of the corresponding algebraic system of equations.

A set of solutions to the boundary value problem under consideration has been found.

Boundary value problems are considered when the number of boundary conditions is greater than the number of unknowns of the system of dynamic equtions and vice versa.

For each of these cases, the solvability conditions of the boundary value problem under consideration are established and its solutions are found.

The results presented are based on the results obtained earlier in [1, 2].

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Synthesis of optimal control for a wave equation with dissipation

A. V. Sukretna[1]

[1]Taras Shevchenko National University of Kyiv, Kyiv, Ukraine

sukretna.a.v@knu.ua

The problem of constructing optimal control in the form of feedback (or synthesis) for distributed systems in a closed form, on the one hand, requires a solution from the practical point of view, and on the other hand, cannot be solved in the general case. The practical benefit of such tasks is quite clear, since feedback control allows to carry out flexible control of various technological processes, and this control can be chosen as optimal from the point of view of one or another criterion, and the presence of a closed formula for control allows for the implementation of control influence in real time. As for the possibility of obtaining such controls from a mathematical point of view, unfortunately, the existing mathematical methods, which are based on the dynamic programming method of R. Bellman and the method of variational inequalities of J.L. Lions, allow bringing the problem to the final formula of optimal synthesis only in some cases. Thus, when applying the dynamic programming method, we come to boundary value problems for Riccati-type equations, and the variational inequality method allows us to construct the necessary optimality conditions in terms of the direct and adjoint problems, leaving out of our attention the ways of solving the resulting system. In light of the above, the construction of the optimal synthesis law in a form reduced to a formula even for a separate class of optimal control problems for a distributed system is a relevant problem that is solved in this work for the optimal control problem, in which the controlled process is described by a boundary value problem for the wave equation with dissipation, and the quality criterion is semi-definite:

$$\begin{cases} y^{\varepsilon}_{tt}(x,t) + 2\gamma y^{\varepsilon}_{t}(x,t) = \mathtt{div}\Big(a^{\varepsilon}(x)\overrightarrow{\mathtt{grad}}\big(y^{\varepsilon}(x,t)\big)\Big) + g^{\varepsilon}(x)v(t), \quad x \in \Omega \subset \mathbb{R}^n, \quad t \in (t_0,T); \\ y^{\varepsilon}(x,t) = 0, \quad x \in \partial\Omega, \quad t \in [t_0,T]; \\ y^{\varepsilon}(x,t_0) = \varphi^{\varepsilon}_0(x), \quad y^{\varepsilon}_t(x,t_0) = \varphi^{\varepsilon}_1(x), \quad x \in \Omega; \end{cases}$$

$$v \in U = \Big\{ v \in L_2(t_0,T) : \ |v(t)| \leqslant \xi \ \text{ ì.ñ. íà } \ [t_0,T] \Big\};$$

$$J(v) = \alpha\Big(\int_{\Omega} q^{\varepsilon}_0(x) y^{\varepsilon}(x,T)dx - \psi_0 \Big)^2 + \beta\Big(\int_{\Omega} q^{\varepsilon}_1(x) y^{\varepsilon}_t(x,T)dx - \psi_1 \Big)^2 + \int_{t_0}^{T} v^2(t)dt \to \inf .$$

Here $a^{\varepsilon} = a^{\varepsilon}(x)$ – is a measurable symmetric matrix of dimension $n \times n$ that satisfies the conditions of uniform ellipticity and boundedness, $g^{\varepsilon}, \varphi^{\varepsilon}_1, q^{\varepsilon}_0, q^{\varepsilon}_1 \in L_2(\Omega)$, $\varphi^{\varepsilon}_1 \in H^1_0(\Omega)$, $\alpha, \beta \geqslant 0$, $\alpha + \beta \neq 0$, $\gamma > 0$, $\psi_0, \psi_1 \in \mathbb{R}$, $t_0 \geqslant 0$, $T > t_0$ – arbitrary fixed points of time.

For the considered problem, in the case when the control does not reach the restriction, the law of optimal synthesis is constructed in an explicit form. However, the coefficients of this synthesis are expressed in terms of infinite series, which is not convenient for practical calculations in real time. In addition, the presence of the parameter ε, which can enter the boundary value problem irregularly, can also significantly complicate these calculations. Therefore, the problem of constructing an approximate control with feedback that would implement a close (in the sense of the quality criterion) behavior of the controlled system naturally arises, therefore the law of approximate averaged synthesis is also proposed and substantiated.

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Genetic algorithms for numerical analysis problems

E. S.Vakal
Taras Shevchenko National University of Kyiv, Kyiv, Ukraine
evhen.vakal@knu.ua

A genetic algorithm (GA) is a modern powerful tool for solving multiparametric optimization problems. The idea of GA consists in computer organization of an evolutionary process of creation, modification and selection of the best solutions (in terms of GA – individuals) aimed at obtaining new better options of problem solution. Each individual in GA is encoded in the form of a string (a chromosome). Each chromosome is composed of genes which contain information on the relevant parameters of solution.

GA starts with an initial population of randomly generated chromosomes. At each iteration, three basic operations – selection, crossover, and mutation – are applied over the current population to yield a new population of chromosomes. This cycle is repeated until some termination criterion is achieved, at which time the best chromosome achieved is generally taken as the solution to the optimization problem. The value of fitness function is the quality estimation of solution encoded in genome. This basic framework can be modified to specifically address the problem of interest.

The continuous and the binary techniques are the main types of genetic algorithms that could be employed for an optimization problem. In most cases the continuous GA, which uses the direct representation of chromosome genes in the form of real numbers, is inherently faster than the binary GA, because the chromosomes do not have to be decoded prior to the evaluation of the fitness function.

The effectiveness of GA has been demonstrated in solving various problems from scientific fields. We used the continuous GA for solving boundary value problems for ordinary differential equations, for differential equations of elliptic and hyperbolic types, and for Cauchy problems.

We also used GA to solve Fredholm integral equations of the second kind, namely, to approximate kernel of the integral equation by sums of a finite number of products of functions of one variable and to obtain optimal parameters of an approximate solution of the linear integral equation by minimizing its integral residual.

In addition, we used the genetic algorithm and a differential evolution algorithm, which is one of the possible "continuous" modifications of GA, to solve some numerical analysis problems, namely, to seek approximate solutions of inconsistent overdetermined systems of transcendental equations with using different norms of residual errors, to compute optimal knots for piecewise minimax approximations by polynomials and fractional rational functions; to seek coefficients and errors of minimax approximations by nonlinear one-variable functions, to obtain the best uniform approximations of many-variables functions, to find optimal parameters of empirical formulas of several variables, to seek optimal knots of minimax approximations by polynomial splines with free knots, to compute coefficients and errors of minimax approximations by polynomial splines with fixed knots.

Genetic algorithms have some advantages: they search for solution directly from a whole set of points – population of individuals, use only the values of fitness function without any information on a function behavior, are relatively resistant to hinting local optima and simple in implementation.

Comparison of ergodic estimates for parameters of mixed fractional Brownian motion in the absence and presence of a trend

M. S. Yakovliev
Taras Shevchenko National University of Kyiv, Kyiv, Ukraine
mykyta.yakovliev@knu.ua

We consider the following mixed fractional Brownian motion with trend

$$X_t = \theta t + \sigma W_t + \kappa B_t^H, \qquad t \in [0, T], \tag{1}$$

where W is a Wiener process, B^H is a fractional Brownian motion with Hurst index $H \in (0, 1)$, B^H is independent of W. And we consider the case without trend ($\theta = 0$):

$$X_t = \sigma W_t + \kappa B_t^H, \qquad t \in [0, T]. \tag{2}$$

Our goal is to compare ergodic estimates for the parameters of those two models. Following [1], we introduce the next four statistics:

$$\phi_N := \frac{X_{Nh}}{N} = \frac{1}{N}\sum_{k=0}^{N-1}\left(X_{(k+1)h} - X_{kh}\right), \quad \xi_N := \frac{1}{N}\sum_{k=0}^{N-1}\left(X_{(k+1)h} - X_{kh}\right)^2,$$

$$\eta_N := \frac{1}{N}\sum_{k=0}^{N-1}\left(X_{(k+1)h} - X_{kh}\right)\left(X_{(k+2)h} - X_{(k+1)h}\right),$$

$$\zeta_N := \frac{1}{N}\sum_{k=0}^{N-1}\left(X_{(k+2)h} - X_{kh}\right)\left(X_{(k+4)h} - X_{(k+2)h}\right),$$

and consider the following estimators for the parameters $(\theta, H, \sigma^2, \kappa^2)$:

$$\hat{\theta}_N = \frac{\phi_N}{h}, \quad \widehat{H}_N = \frac{1}{2}\log_{2+}\frac{\zeta_N - 4\phi_N^2}{\eta_N - \phi_N^2},$$

$$\hat{\kappa}_N^2 = \frac{\eta_N - \phi_N^2}{h^{2\widehat{H}_N}\left(2^{2\widehat{H}_N - 1} - 1\right)}, \quad \hat{\sigma}_N^2 = \frac{1}{h}\left(\xi_N - \phi_N^2 - \hat{\kappa}_N^2 h^{2\widehat{H}_N}\right)$$

The provided estimates are strongly consistent for both models, but the asymptotic normality for model (1) holds with $H \in (0, \frac{1}{2})$ [2] and for model (2) holds with $H \in (0, \frac{1}{2}) \cup (\frac{1}{2}, \frac{3}{4})$ [3]; therefore, the model with trend is not a direct generalisation of the model without trend.

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Robust stability for reaction-diffusion systems without uniqueness

T. V. Yusypiv[1]
[1]Taras Shevchenko National University of Kyiv, Kyiv, Ukraine
taras.yusypiv@knu.ua

In a bounded domain $\Omega \subset \mathbb{R}^n$ we consider the following Reaction-Diffusion system

$$\begin{cases} y_t = a\Delta y - f(y) + h(x) + d, \quad x \in \Omega,\ t > 0, \\ y|_{\partial\Omega} = 0, \end{cases} \tag{1}$$

where $y = y(t,x) = (y^1(t,x), ..., y^N(t,x))$ is unknown vector-function, $f = (f^1, ..., f^N)$, $h = (h^1, ..., h^N)$ are given functions, a is real $N \times N$ matrix with positive symmetric part $\frac{1}{2}(a+a^*) \geqslant \mu I$, $\mu > 0$, $d = (d^1, ..., d^N)$ is an external disturbances.

We assume that $h \in (L^2(\Omega))^N$, and there exist constants $C_1, C_2, C_3, C_4 > 0$, $\gamma_i > 0$, $p_i \geqslant 2$, for $i = \overline{1,N}$, and $g(y) = (g^{ij})_{i,j=1}^N$ (a given matrix-valued function), such that for all $v \in \mathbb{R}^N$ the following properties hold:

$$\sum_{i=1}^N |f^i(v)|^{\frac{p_i}{p_i-1}} \leqslant C_1(1 + \sum_{i=1}^N |v^i|^{p_i}),\ \sum_{i=1}^N f^i(v)v^i \geqslant \sum_{i=1}^N \gamma_i |v^i|^{p_i} - C_2, \tag{2}$$

$$\|g(v)\|^2 := \sum_{i,j=1}^N |g^{ij}(v)|^2 \leqslant C_3,\ Df(v) \geqslant C_4. \tag{3}$$

Let $H = (L^2(\Omega))^N$, $\|d\|_\infty = \operatorname*{ess\,sup}_{t\geqslant 0} \|d(t)\|_H$, Θ is a global attractor generated by semigroup $S : \mathbb{R}_+ \times H \mapsto H$, $S(t, y_0) = \{y(t)|\ y(\cdot)$ is a global weak solution of undisturbed case ($d \equiv 0$) of 1 such that $y(0) = y_0\}$, $\|y\|_\Theta = \inf_{\theta\in\Theta} \|y - \theta\|_H$ and well-known function comparison classes: $\mathcal{K} = \{\gamma \in \mathbb{C}(\mathbb{R}_1) | \gamma(0) = 0, \gamma \text{ is strictly increasing}\}$, $\mathcal{KL} = \{\beta \in \mathbb{C}(\mathbb{R}_+ \cdot \mathbb{R}_+) | \beta(\cdot, t) \in \mathcal{K}, \beta(s, \cdot) \searrow 0\}$.

Theorem. *The problem (1):*
(i) (see [1]) *with* $f \in \mathbb{C}(\mathbb{R}^N; \mathbb{R}^N)$, $d = d(t,x) \in L^\infty(\mathbb{R}_+, H)$ *under condition* (2) *satisfies the asymptotic gain (AG) property with respect to* Θ*, i.e., there exist* $R_0 > 0$*, and* $\gamma \in \mathcal{K}$ *such that* $\forall y_0 \in H$, $\forall \|d\|_\infty \leqslant R_0$

$$\overline{\lim_{t\to\infty}} \|y(t, y_0, d)\|_\Theta \leqslant \gamma(\|d\|_\infty);$$

(ii) (see [2]) *with* $f \in \mathbb{C}^1(\mathbb{R}^N; \mathbb{R}^N)$, $d = g(y)d(t)$, $d(t) \in L^\infty(\mathbb{R}_+, H)$ *under conditions* (2), (3) *satisfies the locally ISS with respect to* Θ*, i.e., there exist* $r > 0$, $\beta \in \mathcal{KL}$*, and* $\gamma \in \mathcal{K}$ *such that* $\forall \|y_0\|_H \leqslant r$, $\forall \|d\|_\infty \leqslant r$

$$\|y(t, y_0, d)\|_\Theta \leqslant \beta(\|y_0\|_\Theta, t) + \gamma(\|d\|_\infty),\ t \geqslant 0.$$

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Taras Shevchenko National University of Kyiv

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ABSTRACTS

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