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ABSTRACTS

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Contents

Properties and calculation of projection coefficients for Gaussian-Volterra process with a linear kernel

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Based on the joint research with Yuliya Mishura.

Let W be a Wiener process and $\Delta_i = X_i - X_{i-1},\ i \geqslant 1$, where X_t is the Gaussian-Volterra process with a linear kernel, i.e.,

$$X_t = \int_0^t (t-s)\, dW_s.$$

We solve the problem of projection of Δ_1 on n subsequent increments $\Delta_i,\ 2 \leqslant i \leqslant n+1$. Namely, we obtain the explicit formulas for the coefficients $\{c_n^{(k)},\ 2 \leqslant k \leqslant n+1\}$, such that by theorem of normal correlation,

$$\mathsf{E}(\Delta_1 | \Delta_2, \dots, \Delta_{n+1}) = \sum_{k=2}^{n+1} c_n^{(k)} \Delta_k. \tag{1}$$

The similar problem for fBm B_t^H was studied in [1]. The following hypothesis was investigated there.

For $H > 1/2$ all projection coefficients are strictly positive.

This hypothesis was checked in [1] numerically and analytically for small n. However, it was not proved analytically. The projection problem is also still unsolved for the general Gaussian-Volterra process.

We came up with the idea to investigate properties of the simplest Gaussian-Volterra process, determine the projection coefficients, and try to understand how the properties of the process affect the properties of the coefficients.

In particular, it was established that the coefficients from (1) change their sign. But at the same time, the increments of X_t in the case of non-overlapping intervals are positively correlated. Note, that fBm B_t^H also has positively correlated increments. Thus, we can conclude that a positive correlation of increments does not guarantee the positivity of the projection coefficients.

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The word problem of small complexity

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Let G be a finitely generated group, and S be a finite generating set of G. The word problem in G with respect to S is the formal language $\mathrm{WP}(G, S) = \{w \in (S \cup S^{-1})^* : w =_G e\}$. The word problem is a central algorithmic problem in group theory. It was studied for many classes of groups and with respect to various computational models.

In my talk, I will consider the time complexity of the word problem with respect to deterministic Turing machines with single and multiple tapes. The connection between the word problem and the growth of groups will be presented. The growth function $\gamma_S(n)$ of the group G counts the number of group elements represented by words over $S \cup S^{-1}$ of length $\leqslant n$. By Gromov's celebrated result, the growth function of a group G is bounded by a polynomial if and only if G is virtually nilpotent, that is, it has a nilpotent subgroup of finite index. The word problem for virtually nilpotent groups can be solved in time $o(n^2)$ by a single-tape Turing machine, while the word problem for groups of exponential growth requires at least quadratic time. The following question naturally arises:

Question. *Let G be a finitely generated group, and the word problem in G is solvable in time $o(n^2)$ by a single-tape Turing machine. Is G virtually nilpotent?*

The question can be rephrased as follows: does there exist a group G of intermediate growth between polynomial and exponential with the word problem solvable in time $o(n^2)$ by a single-tape Turing machine. The well-known examples of groups of intermediate growth are generated by bounded Mealy automata (bounded in terms of Sidki [2]). Bounded automaton groups belong to the class of contracting groups, where the word problem is solvable in polynomial time. We prove that linearithmic time is sufficient for multi-tape Turing machines.

Theorem. *Let G be a group generated by a bounded automaton. The word problem in G is solvable in time $O(n \log n)$ by a multi-tape Turing machine. If G is strongly contracting, then the word problem in G is solvable in time $O(n)$ by a multi-tape Turing machine and in time $O(n^2)$ by a single-tape Turing machine.*

Bounded automata are polynomial automata of zero degree. In [1], it is proved that for groups generated by polynomial automata, the word problem is solvable in subexponential time. A more careful analysis gives quasilinear time.

Theorem. *Let G be a group generated by a polynomial automaton. The word problem in G is solvable in time $O(n(\log n)^d)$ by a multi-tape Turing machine for some $d \geqslant 0$.*

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Application of Genetic Algorithms in Assisted Relaying Network

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New advances in the practical application of Unmanned Aerial Vehicles (UAVs) and 5G present the concept of utilizing multiple Terminal Devices (TDs). This emerging field of practical methods drives the advancement of Mobile Edge Computing (MEC), which accelerates computations by leveraging the computational capacity of TDs at network edges.

The UAVs are assisting computation as MEC servers for TDs, while also delivering a portion of tasks to ground APs for remote execution. The technology of UAV-assisted relay in MEC emerges as an approach to extend network coverage [1,2]. Here, ground users delegate their computing tasks to airborne UAVs, which can then offload some of these tasks to remote APs for computation.

The versatility of the problem's formulations leads to different natures of the proposed solutions, heavily dependent on the initial formulation of the problem, and thus lacking in universality.

This research presents a resource allocation problem, specifically the minimization of task completion time within the network, achieved by optimizing communication bandwidth, UAV transmit power, computation resources, task partitioning, and UAV's three-dimensional (3D) location deployment. The general formal definition of the problem for resource allocation in MEC is presented. For the given general formulation, we propose to solve it using a genetic algorithm approach. Other approaches to solving this problem are also discussed.

A practical implementation comparison of this method with one of the state-of-the-art methods for a particular case, described in [3] but with changed constraints, is shown. This non-convex problem could be solved by the Successive Complex Approximations (SCA) method.

To increase its generality, the constraints of the described problem are hardened by changing fading to the block-length method, and the Rician fading model is used for the UAV-ground channels. An analysis of the modified problem is conducted to check the applicability of the SCA-based method on the consistency and corresponding modifications for the above transformations are made.

Both the proposed genetic algorithm and the modified SCA-based approach are implemented, and their efficiency is compared. The analysis of the results shows that the effectiveness of the genetic algorithm is not worse than the referenced one.

Acknowledgements

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On monotonicity of Shannon and Rényi entropies of the Poisson distribution as the functions of intensity parameter

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For a discrete distribution $\{p_i, i \geqslant 1\}$ its Shannon entropy is defined as

$$H_S(p_i, i \geqslant 1) = -\sum_{i \geqslant 1} p_i \log p_i$$

and its Rényi entropy is defined as

$$H_R^{\alpha}(p_i, i \geqslant 1) = \frac{1}{1-\alpha} \log \left(\sum_{i \geqslant 1} p_i^{\alpha} \right), \ \alpha > 0, \ \alpha \neq 1.$$

We consider Poisson distribution with parameter $\lambda > 0$:

$$p_k(\lambda) = \mathsf{P}\{\xi_\lambda = k\} = \frac{\lambda^k e^{-\lambda}}{k!}, \ k \in \mathbb{N} \cup \{0\}$$

and study the properties of its Shannon and Rényi entropies

$$H_S(\lambda) = -\sum_{k=0}^{\infty} \frac{\lambda^k e^{-\lambda}}{k!} \log \left(\frac{\lambda^k e^{-\lambda}}{k!} \right), \quad H_R^{\alpha}(\lambda) = \frac{1}{1-\alpha} \log \left(\sum_{k=0}^{\infty} \left(\frac{\lambda^k}{k!} e^{-\lambda} \right)^{\alpha} \right).$$

Theorem. *Functions $H_S(\lambda)$ and $H_R^{\alpha}(\lambda)$ for every fixed $\alpha > 0$, $\alpha \neq 1$, are strictly increasing on $(0, +\infty)$.*

The proof is based on application of Karamata's inequality [2] to the terms of Poisson distribution. The verification of conditions of this inequality is nontrivial. It involves rearrangement of the terms of Poisson distribution into a non-increasing sequence and exploring the monotonicity of partial sums of this sequence.

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Life path and scientific-pedagogical activity of Heorhii Mykolaiovych Polozhii

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April 23 marked the 110th anniversary of the birth of the famous mathematician, Corresponding Member of the Academy of Sciences of the Ukrainian SSR, Doctor of Physical and Mathematical Sciences, Professor Heorhii Mykolaiovych Polozhii, who worked for almost 20 years at the Faculty of Mechanics and Mathematics of Taras Shevchenko Kyiv National University.

H.M. Polozhii headed the Department of Mathematical Physics (1951 – 1958), later, at the Faculty of Mechanics and Mathematics, he founded the Department of Computational Mathematics and was its first head (1958 – 1968). A computing center was created at Kyiv University with his active participation. H.M. Polozhii founded a scientific school on the theory of generalized analytical functions at Kyiv University. He trained 22 candidates of sciences, six of whom defended their doctoral theses. Among his pupils are academicians of the National Academy of Sciences of Ukraine I.I. Lyashko and V.L. Makarov, Corresponding Member of the National Academy of Sciences of Ukraine B.M. Bublyk, professors of Kyiv University A.A. Hlushchenko, O.O. Kapshyvyi, I.M. Lyashenko. His pupils I.I. Lyashko, N.O. Pakhareva, A.A. Hlushchenko and O.O. Kapshyvyi headed the Department of Mathematical Physics at one time, and later I.I. Lyashko headed the Department of Computational Mathematics. Polozhii's pupils became outstanding scientists and, in turn, passed on the baton of an outstanding scientist to their pupils.

Heorhii Mykolaiovych wrote the first textbook on mathematical physics in Ukrainian, published one of the first textbooks on computational mathematics for USSR universities, which was later republished in German and Polish. H.M. Polozhii are the author of 114 scientific works, including monographs, which have been republished in English and German. H.M. Polozhii was the founder of such two new scientific fields:

1. theories of generalized p- and (p,q)-analytic functions of a complex variable and their applications in the theory of boundary value problems of mathematical physics and mechanics of continuous media;

2. the method of summary representation and P-transformations for the numerical solution of boundary value problems of mathematical physics.

Scientific results of H.M. Polozhii found wide application in the study of complex applied problems, in particular, the mechanics of solid media and the theory of filtration.

Until the last days of his life, H.M. Polozhii remained a man of exceptional energy and diligence, principled and demanding both of himself and his pupils, but at the same time he remained an attentive teacher and a true friend. Heorhii Mykolaiovych lived only 54 years. During this time, he managed to create a scientific school, two new scientific fields, write a number of monographs and tutorials, and create a new department. The life of H.M. Polozhii is an example of service to Science and the Motherland.

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Mathematical models of real economic and social processes

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The mathematical models have a practical orientation and are built on an interdisciplinary basis and aimed at the study, analysis, research and forecasts of real economic and social processes. The results are published in periodical scientific publications included in the list of specialized publications of Ukraine, in scientific and metric databases, in particular Scopus, Web of Science [1-7]. Various quantitative methods of analysis of international economic and social relations were used in the construction of the models. In particular, the algorithms of correlation analysis, mean analysis, variance analysis, linear and non-linear regression analysis, cluster analysis and optimization methods were used.

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Sandwich Semigroups Isomorphisms

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For an arbitrary but fixed element $a \in S$ of a semigroup $(S, \cdot)$ we can define a new operation by the next equality

$$x *_a y = x \cdot a \cdot y, \quad \text{for any elements } \ x,\ y \in S.$$

The operation $*_a$ is called a *sandwich-multiplication*, and the semigroup $(S, *_a)$ is called a *variant* or a *sandwich-semigroup* with the *sandwich element* a.

Naturally arises a question when variants are isomorphic. We studied this question for variants of a commutative band with zero.

The *band* is a semigroup S each element of it is idempotent. This semigroup is naturally partially ordered, thus $a \leqslant b$ if and only if $ab = ba = a$.

We call an interval of a commutative band the set

$$S_{[a,b]} = \{x \in S | x \cdot a = a,\ b \cdot x = x\}.$$

Let a be a fixed element of a commutative band S with zero. For each element x from the interval $S_{[0,a]}$ by $\Omega(x)$ we denote a set $\{y \in S | a \cdot y = x\}$. A weight $\omega(x)$ of an element x we call a power of this set $\omega(x) = |\Omega(x)|$.

Theorem 1. *Two variants $(S, *_a)$ and $(S, *_b)$ of a commutative band S with zero are isomorphic if and only if there exists an isomorphism from the interval $S_{[0,a]}$ to the interval $S_{[0,b]}$, which preserves weights of all elements.*

Theorem 1 was used to classify variants of various semigroups, a proof of the theorem and some easy examples could be found in the paper [1]. We used this theorem to classify variants of other more interesting lattices, for example, for the lattice of partitions of a countable set in [2].

The question which semigroups could not be isomorphic to a sandwich semigroup is also arises. Obviously these semigroups should not contain an identity, since any monoid is a variant of itself generated by identity as a sandwich element. We studied a question when Brandt semigroup is not a variant.

A semigroup S with zero is called a *Brandt semigroup* if $eSf \neq 0$ for any non-zero idempotents e, f, and for any $a \neq 0$ there exists unique element e, such that $ea = a$, unique element f, such that $af = a$, and unique element a', such that $a'a = f$.

The bicyclic semigroup is the semigroup $\mathscr{C}(p, q)$ with identity element generated by two symbols p and q subject to the single generating relation $pq = 1$, thus $\mathscr{C}(p, q) = \langle p,\ q \,|\, pq = 1 \rangle$.

Theorem 2. *Let S be a semigroup which does not contain a bicyclic subsemigroup. Then for any $a \in S$ the variant $(S, *_a)$ is not a Brandt semigroup.*

Theorem 3. *A finite Brandt semigroup is not a variant.*

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The Method of Summary Representation and its significance in the solution applied problems of mathematical physics

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One of the many areas of scientific activity of the corresponding member of the National Academy of Sciences of Ukraine, Heorhiy Mykolayovych Polozhii, was the Method of Summary Representation for solving problems in mathematical physics and technology. Starting with the first publications in 1960, and especially after the publication of the fundamental monograph [1], this method turned not only into a new scientific direction, but also into a powerful, easy-to-use tool for solving difficult applied problems.

The Method of Summary Representation and Π-transformations is a discrete analogue of a set of methods, in particular, integral images, Green's functions, integral transformations, separation of variables, as well as the theory of special functions of a discrete argument. The characteristic features of this numerical analytical method are as follows:

1) the solution of an elliptic type problem in an arbitrary node of the grid area is written in the form of an analytical formula of Summary Representation (FSR), which is explicit or contains a small number of unknown parameters;

2) it is possible to solve boundary-value problems in regions with boundary geometry that differs little from inscribed rectangles;

3) the method is extended to three-dimensional problems of the elliptic type, to the biharmonic equation; on equations of parabolic, hyperbolic type (one-dimensional and two-dimensional) with constant and variable coefficients [1];

4) in further studies, FSR was obtained for unlimited regions;

5) the use of the method made it possible to solve new spectral problems for second-order equations in ordinary and partial finite differences;

6) with the help of this method, important applied problems in the theory of filtration, theory of elasticity, theory of plates and shells, theory of potential were solved. To illustrate the main stages of the Method Summary Representation and Π-transformation, we consider obtaining the first FSR for the boundary value problem [1]

$$\Delta u - 2c * u(x,y) = -f(x,y), \qquad (x,y) \in G; \tag{1}$$

$$\overline{G} = G \cup \partial G \equiv \{(x,y) \in [0,a] \times [0,b]\}; \tag{2.h}$$

$$u(x,0) = \mu_1(x), \quad u(x,b) = \mu_3(x), \quad x \in [0,a];$$

$$u(a,y) = \mu_2(y), \quad u(0,y) = \mu_4(y), \quad y \in [0,b). \tag{2.v}$$

The formula for Summary Representation is obtained [1]

$$\tilde{u}(x_i) = P_1 \left(\Phi(i)\tilde{A} + \Psi(i)\tilde{B} - \sum_{k=1}^{i-1} G(i-k) P_1^{\top} \tilde{H}(x_k) \right), \quad i = 1, \dots, \overline{m}, \tag{4}$$

where $\Phi(i), \Psi(i), \mathcal{G}(\mathcal{K})$ are the known diagonal matrices (denoted through the input data), and conditions $(2.v)$ are used to explicitly find the vectors $\vec{A}, \vec{B}$.

The numerical experiment for the model problem (1)-(2) demonstrated how the Method of Summary Representation actually works.

The numerical experiment for the model problem (1)-(2) demonstrated how the Method of Summary Representation actually works.

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MINIMAL TILED ORDERS

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Let $\Lambda = \{\mathcal{O}, \mathcal{E}(\Lambda) = (\alpha_{ij})\}$ be a reduced tiled order on a discrete valuation ring $\mathcal{O}$ with unitary maximum ideal $\mathbf{m} = \pi\mathcal{O}$, where π is a prime element. V.A. Jategaonkar gave necessary and sufficient conditions, that $\Lambda/\pi\Lambda$-module $M/\mathbf{m}M$ decomposes into direct sum of modules for an irreducible Λ-lattice M. Herewith projective dimension of lattice M is equal to infinity.

Lets say that tiled order Λ satisfies Jategaonkar's condition, if such irreducible Λ-lattice doesn't exist, that $\Lambda/\pi\Lambda$-module $M/\mathbf{m}M$ decomposes into direct sum of modules.

According to tiled order Λ we build a simple graph $G = G(\Lambda)$: set of vertices of a graph matches with set of vertices of quiver Q. Points i and j are connected with an edge if and only if $\alpha_{ij} + \alpha_{ji} = 1$. If the graph $G(\Lambda)$ is connected, then $d = \sum_{i,j=1}^{n} \alpha_{ij} \leqslant \frac{(n-1)n(n+1)}{6}$.

Theorem 1. *Let Λ be reduced tiled order with exponent matrix $\mathcal{E}(\Lambda) = (\alpha_{ij}) \in M_n(\mathbb{Z})$ satisfies Jategaonkar's condition and graph $G(\Lambda)$ has two connected components. Then $d = \sum_{i,j=1}^{n} \alpha_{ij} \leqslant \frac{(n-1)n(n+1)}{6}$.*

Theorem 2. *Let Λ be reduced tiled order with exponent matrix $\mathcal{E}(\Lambda) = (\alpha_{ij}) \in M_n(\mathbb{Z})$, I be an ideal of order Λ with the exponent matrix $\mathcal{E}(I) = (\kappa_{ij})$ and $M = (\alpha_1, \ldots, \alpha_n)$ be an irreducible Λ-lattice. Λ/I - module M/MI is decomposed into a direct sum, if and only if there exists a partition of the set of indices $I = \{1, \ldots, n\} = I_1 \cup I_2$, $I_1 \cap I_2 = \varnothing$ such that for all i, k belonging to different subsets the inequality $\alpha_i + \alpha_{ik} \geqslant \min_{1\leqslant j\leqslant n}(\alpha_j + \kappa_{jk})$ holds.*

A tiled order A with the exponent matrix $\mathcal{E}(A) = (a_{ij})$ and with the quiver Q is called minimal if there is no non-isomorphic tiled order B with the exponent matrix $\mathcal{E}(B) = (b_{ij})$ and the quiver Q such that $b_{ij} \leqslant a_{ij}$ for all i, j.

Consider a permissible quiver with the adjacency matrix

$$[Q] = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}.$$

This quiver is the quiver of exactly two non-isomorphic tiled orders. Both tiled orders are minimal. The global dimension of one order is finite, and the second is equal to infinity.

Acoustic Radiation Force in the Liquid with Bodies

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Study of interaction between acoustical field and small particles lo-cated in liquid is of great intetrest. They are widely used in different technological applications ranging from medical equipment production to chemical industry. Acoustical pressure acting upon a particle manifests itself in a way that sound pulse averaged over the period of incident wave varies in some volume. This causes the time independent stationary components of sound pressure influencing the particle in the volume and results in a corresponding force referred to as acoustic radiation force (ARF). ARF enables one to control the particles in the liquid, perform, for instance, their sorting by size, mixing, coagulation of the carrying agent, acoustic cleaning, degassing, acoustic cavitation control etc [1].

Development of the analytical approach and systematic investigation of the anomalous features of acoustic diffraction and scattering processes as well as acoustic radiation forces is performed with accounting for influence of different factors [1,2]. Specific singularities of an analytical solution of this class of the hydroelasticity problems is caused by presense of the cavity surfaces with a fluid and submerged particle that geometrically belong to different frames. It leads to necessity of rebuilding of solution of the Helmholtz equation obtained with respect to a cylindrical frame (related to the cavity) in spherical frame (related to the particle) and vice versa. As a result, solution of the infinite systems of the algebraic equations with respect of the sought-of expansion coefficients have to be built. The second source of complication is the necessity of application of translational addition theorems of higher transcendental functions [1].

Both solid wall (solid cavity) and compliant wall vessels as well as elastic cylindrical shell filled with liquid were studied. It can be of either infinite or half infinite length. The different types of submerged bodies were investigated: solid, gaseous, liquid, and covered with elastic shell. Both solitary body and system of bodies were considered [2]. It was found the system of bodies located in the cavity or vessel induces the so called secondary acoustic radiation forces caused by the bodies' interaction. Those forced were determined as well. The approach proposed comprises the mechanical model, mathematical problem statements, solution procedure which based on the analytical methods of mechanics and mathematical physics as well as calculation technique based on the systems of computer algebra application. The numerical experiment were performed with the use of wide range of input data to reproduce the characteristics of diffraction processes and acoustic radiation forces depending on the frequency (wavelength) of the acoustic loading and physical and mechanical properties of the system under consideration. Depending on the frequency of the incident wave, the radiation force can change its magnitude and direction. As a result, a range of recommendations and guidance for techno-logical utilization of the results obtained to different kinds of applications were proposed.

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Modeling of the active element of the adaptive optical system

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The work summarizes and develops the results obtained earlier in [1-3], where the problems of finding optimal parameters of mechanical devices for excitation and formation of wave motion are investigated. Such devices can be used to generate, transform and transmit information and wave energy. In this work, the problem of modulation of a mirror fixed on active supports is considered. The task is to find the control forces and their characteristics - application points, amplitude and phase of the forces, which provide the best approximation of the given shape and phase of the mirror oscillations, taking into account structural inhomogeneities (defects) with uncertain geometric and mechanical characteristics. The physical properties of defects, their location , size and shape are uncertain. The study consists of deterministic and stochastic parts.

In the deterministic part, the characteristics of the defects were considered as known. It is reduced to the problem of finding of the minimum of the functional, which is built on the root mean square deviation of the deflections of the plate from the given wave profile. The optimization parameters were the application points, amplitudes and phases of the control forces of the active supports [2]. To speed up the solution of the boundary value problem of the plate equilibrium, the solution of which forms the objective function of the optimization problem, the defects were modeled by high-order point singularities, which ensured the accuracy of the given order with respect to a small parameter - the area of the region of inhomogeneity. When solving the deterministic problem, the methods of the generalized Green's function and harmonic analysis were used with control of the number of harmonics during the distribution along the circular coordinate of the plate.

In the stochastic part, it was considered that the number of defects and their characteristics are uncertain or there is partial information about them. Accordingly, the method of stochastic optimization based on the Monte Carlo method was applied. Usually, in reliability theory, risk is quantified by the probability of failure. Although the failure probability is very popular, it has undesirable mathematical properties such as incoherence and discontinuity for sampling distributions. To overcome these shortcomings, a new alternative measure of risk, buffered probability of failure (BPF) was used. The buffered probability of exceedance, bPOE, generalizes BPF in the case where the system failure threshold can be any number (not only 0). These risk measures are based on the properties of the CVaR risk measure. bPOE has exceptional mathematical properties (under general conditions, bPOE is quasi-convex with respect to a random variable)

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Attractors for infinite-dimensional impulsive systems

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The qualitative behavior of dissipative infinite-dimensional continuous dynamical systems is investigated by the methods of the global attractor theory. The main problem we face when we try to expand this theory to impulsive DS

$$\frac{du}{dt} = Au + F(u), \ u \notin M,$$

$$\Delta u|_{u \in M} = Iu - u,$$

where

$$\Delta u(t) = u(t+0) - u(t-0),$$

is the lack of continuous dependence on the initial data. Our approach is based on the notion of uniform attractor, commonly used for non-autonomous problems, in particular, for systems with impulses at fixed moments of time.

Definition. A compact set $\Theta \subset H$ is called uniform attractor of DS V if

$$\text{for every bounded } B \subset H \ \ dist(V(t,B), \Theta) \to 0 \text{ as } t \to \infty,$$

and Θ is minimal among all closed uniformly attracting sets.

Sufficient conditions for existence of uniform attractors and investigation their properties for semilinear PDEs with impulsive perturbations are proposed.

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Optimal control problem for degenerate elliptic variation inequality: existence result

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We investigate the optimal control problem for degenerate elliptic variation inequality with homogeneous Dirichlet boundary conditions in the case when the linear elliptic operator associated with it does not satisfy the conditions of coercivity and boundedness.

Let $\Omega \subset \mathbb{R}^N$ be some nonempty open bounded set, $N > 3$. Let us consider the following optimal control problem

$$I(u,y) = \frac{1}{2}\left\| y - \frac{y_{ad}}{\sqrt{\rho}} \right\|^2_{L^2(\Omega,\rho dx)} + \frac{1}{2}\|u\|^2_{L^2(\Omega,\rho^{-1}dx)} \to \inf, \tag{2}$$

$$u \in U_\partial,\ \ y \in K, \tag{3}$$

$$\langle -\mathrm{div}(\rho(x)\nabla y), v - y\rangle_{W_\rho} \geqslant \langle f(x) + u(x), v - y\rangle_{W_\rho}\ \forall v \in K, \tag{4}$$

- $f \in L^2(\Omega;\rho^{-1}dx)$ and $y_{ad} \in L^2(\Omega)$ are given distributions,
- $L^2(\Omega,\rho dx)$ and $L^2(\Omega,\rho^{-1}dx)$ are weight Hilbert spaces associated with a degenerate weight $\rho \in L^1(\Omega)$ with norms $\|f\|^2_{L^2(\Omega,\rho dx)} = \int_\Omega f^2\rho dx$, $\|f\|^2_{L^2(\Omega,\rho^{-1}dx)} = \int_\Omega f^2\rho^{-1}dx$
- $W_\rho = W_0^{1,2}(\Omega,\rho dx)$ is a set of such functions $y \in W_0^{1,1}(\Omega)$, that $\rho y^2 \in L^1(\Omega)$, $\rho|\nabla y|^2_{\mathbb{R}^N} \in L^1(\Omega)$,
- $U_\partial = \{u \in L^2(\Omega,\rho^{-1}dx) :\ \|u - u_0\|_{L^2(\Omega,\rho^{-1}dx)} \leqslant R\}$, $u_0 \in L^2(\Omega,\rho^{-1}dx)$,
- $\mathcal{W}_\rho$ is a closure of $C_0^\infty(\Omega)$ relative to the norm:

$$\|y\|^2_{\mathcal{W}_\rho} = \int_\Omega y^2\rho dx + \int_\Omega \left|\nabla y + \tfrac{y}{2}\nabla \ln \rho\right|^2_{\mathbb{R}^N} \rho dx,$$

- $K \subset W_\rho \cap \mathcal{W}_\rho$ is a nonempty convex closed subset.

Our aim is an obtaining of sufficient conditions for function ρ, under which the given optimal control problem has a unique solution even if the space of finite functions $C_0^\infty(\Omega)$ is not dense in the weighted Sobolev space $W_0^{1,2}(\Omega,\rho\,dx)$. For this purpose, we consider a transformation, according to which the original problem is reduced to the optimal control problem for an elliptic variation inequality with unbounded coefficients of the potential type, and the issue of the existence of its unique solution is investigated using the Hardy-Poincaré type inequality. Thus, the main result of our investigation is the next theorem [1].

Theorem. *Let $\rho : \Omega \to \mathbb{R}_+$ be a degenerate weight function of potential type. Let $f \in L^2(\Omega,\rho^{-1}dx)$, $y_{ad} \in L^2(\Omega)$ be given functions. Then the optimal control problem (2)-(4) has a unique solution (u^0, y^0) in the space $L^2(\Omega,\rho^{-1}dx) \times \mathcal{W}_\rho$.*

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Optimal Transfers of Space Vehicles with Combination of High- and Low-Thrust Arc

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The main difficulties in manned mission designing are dealt with both severe transfer time limitations and high payload requirements. Bi-modal nuclear thermal rocket (BNTR) propulsion is one of the most promising options of choice to manage these difficulties. The advantages of nuclear thermal rocket engine high thrust value ($10 \div 1000\,$kN) and nuclear electric propulsion high efficiency (specific impulse more than 2000 s) are joined in BNTR where the reactor both generates electrical power using thermionic or closed cycle dynamic conversion system and produces the thrust with the help of hydrogen blown through the core.

It is assumed that the interplanetary trajectory of the BNTR vehicle consists of the low-thrust heliocentric arc where electric thrusters are used and one high-thrust planet centric arc where nuclear thermal rocket engines are used. So, the problem of optimal flyby of the celestial body with the start from initial heocentric orbit is considered. For such a problem the joint optimization of the combination of high- and low-thrust arcs as well as the parameters of high- and low-thrust subsystems of BNTR propulsion system is considered. The criterion of optimization is payload mass. The patched-conics method is applied to reduce the problem to the external (heliocentric) and the internal (planet centric) parts. The high-thrust arcs have been analyzed using impulsive approximation approach. The analytical solution for the low-thrust heliocentric maneuver has been developed using transporting trajectory method. Basing on this solution the final analytical expression for the payload mass was obtained. It is proven, that for all values of electrical power of low-thrust subsystem the inclusion of low-thrust arc is optimal.

The numerical results are obtained for the Hohmann type Earth-Mars transfer. The distribution of Δv (Δv is velocity change) budget between high and low-thrust maneuvers is analyzed. Optimal values of the parameters that define the masses of high- and low-thrust subsystems of BNTR propulsion system have been evaluated. An efficiency of high- and low-thrust combination is analyzed in comparison with fully high-thrust transfer. The possibilities of the application of proposed approaches are discussed.

Non-local differential operators and Markov processes

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For $f \in C_0^2(\mathbb{R}^d)$ consider an operator

$$Lf(x) := b(x)\nabla f(x) + \int_{\mathbb{R}^d} \left(f(x+u) - f(x) - \nabla f(x)u 1_{|u|\leqslant 1}\right) \nu(x, du)$$

where $b(\cdot) : \mathbb{R}^d \to \mathbb{R}$ and $\nu(x,\cdot)$ is a Lévy-type kernel, i.e.

$$\int_{\mathbb{R}^d} (1 \wedge |u|^2)\nu(x, du) < \infty, \quad \forall x \in \mathbb{R}^d.$$

Under certain regularity assumptions on $b(\cdot)$ and $\nu(x,\cdot)$ we show that there exists a solution to the martingale problem for $(L, C_0^2(\mathbb{R}^d))$. We discuss the general methodology of the problem and the existing results.

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Architectural Patterns: The Secret of a Professional Programmer

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Architectural patterns play a crucial role in software development, offering standardized solutions to common problems and fostering code reusability and adaptability. By understanding and implementing these patterns, developers can significantly improve the quality and efficiency of their software products.

In this work, we delve into the history of design patterns and explain how and why they have become game changers in software development. Using a practical example that simulates the operation of a furniture store, we illustrate the significance of architectural patterns, providing insights into how these patterns can enhance the efficiency, understanding, maintenance, and scalability of programs.

The example is presented in two stages:

1. Initial Code: Basic setup managing chairs and tables.

2. Refinement: Introduction of the FurnitureMaker interface for flexible furniture creation and easier extension (e.g., adding sofas). This example demonstrates how development costs can be significantly reduced by leveraging code reusability within a well-structured software architecture.

Conclusion: Architectural patterns provide standardized, reusable solutions, enhancing software quality and adaptability. They allow for a significant reduction in development costs. Understanding and applying these patterns is essential for professional software development.

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Special Periodic Points of Stokes Flow In a Rectangular Cavity

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In recent years, there has been a significant development of interest in the implementation of a qualitative process of mixing in flows in two-dimensional rectangular cavities without the participation of physical mixers in the process itself. This becomes possible when the flow of an incompressible viscous fluid is periodically excited in a rectangular cavity with the help of tangential velocities applied to its moving walls. The results obtained in this direction relate to problems in which the side walls in a rectangular cavity are free from loads, which is physically impossible to implement. The purpose of the research is to build a similar model, which is proposed in article [1], considering the case of fixed side walls in a rectangular cavity. Also, the goal is to find special periodic points of the third order and establish their type.

It is known that the movement of individual flow particles is considered in a given velocity field and is reduced to the solution of advection equations, which represent a system of ordinary differential equations of the first order with a complex functional dependence in the right-hand parts:

$$\frac{dx_i}{dt} = \frac{\partial \psi}{\partial y}, \frac{dy_i}{dt} = -\frac{\partial \psi}{\partial x}, \tag{1}$$

whith initial conditions $x_i(t_0) = x_{i0}$, $y_i(t_0) = y_{i0}$, $i = \overline{1, n}$.

The flow function ψ is a solution of a biharmonic problem, a detailed description of the construction of the solution of which is described in the article [2]. The resulting solution defines the velocity field, that is, the right-hand sides of the advection equations (1).

Important in studying the advection of a passive non-inertial particle is the knowledge of the periodic points of the process of order p, that is, such initial conditions in the advection equation (1), when the point accurately returns to its initial position in p periods. A fundamental element of the analysis of the advection process is the classification of periodic points into elliptical and hyperbolic.

We will classify the type of periodic point analytically by determining the eigenvalues λ_1 and λ_2 of the Jacobian matrix of the linearized system (1) in the vicinity of the considered point. If λ_1 and λ_2 are complex conjugate, the periodic point has an elliptic type. If λ_1 and $\lambda_2 = \frac{1}{\lambda_1}$ are real, the periodic point is of hyperbolic type. There can also be a situation when $\lambda_1 = \lambda_2 = \pm 1$, which corresponds to the degenerate case where the periodic point is parabolic: in this case, any small change in the velocity field causes the periodic point to become elliptical or hyperbolic.

The Jacobian elements of the matrix M are calculated by solving system (1) for four initial conditions $(\overline{x} + \epsilon, \overline{y})$, $(\overline{x} - \epsilon, \overline{y})$, $(\overline{x}, \overline{y} + \epsilon)$, $(\overline{x}, \overline{y} - \epsilon)$ where $(\overline{x}, \overline{y})$ are the rectangular coordinates of a periodic point, and ϵ is an arbitrarily small value.

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Application of physically informed neural networks to solve boundary value problems in mechanics

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A general algorithm for constructing solutions to the problems of mechanics of deformable solids using physically informed neural networks (PINNs) is considered. The proposed approach makes it possible to reduce the boundary value problems of deformable solid mechanics to the corresponding optimization problems, and the use of physically informed neural networks within the framework of the considered approach reduces the solution of a wide class of problems of deformable solid mechanics to the construction of substitution functions [1], which depend on the boundary and initial conditions of the problem and the neural network solution of the corresponding optimization problem. The objective of this work is the construction of an algorithm for solving a plane problem of elasticity theory using physically informed neural networks and to determine the suitability of this algorithm for numerical solution of dynamic and static problems of deformable solid mechanics, in particular, a plane problem of non-homogeneous elasticity theory.

To illustrate the proposed technique, the number of mechanical problems (both one and two dimensional, and static as well as dynamic) were considered.

Namely, the problems of calculating stress-strain state for a one-dimensional dynamic problem of longitudinal vibrations of a rod, one-dimensional dynamic problem of heating the rod, and two-dimensional problem of static elasticity were considered.

Also the problem of non-homogeneous plane elasticity was analysed with the help of physics informed neural networks methodology.

The proposed methodology can also be extended to the case of a three-dimensional problem of deformable solid mechanics, as well as for inhomogeneous media.

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Investigation of the vesicular sound nature of human breathing

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We used traditional and original methods to record vesicular and tracheal breathing sounds of a normal person, obtained their spectral and fractal analysis and proved that the sounds have a multifractal character and the nature of the occurrence of vesicular and tracheal sounds is different. It was shown that vesicular sound occurs as a result of stretching-compression deformation of the lung parenchyma during breathing, and tracheal sounds are known to occur due to pressure pulsations on the inner surface of the trachea. These pulsations are caused by the unsteadiness of the air flow in the area of the glottis.

A mathematical model to the process of generation of vesicular breathing sound, based on the assumption on a significant contribution of the vibration of the alveoli walls to the production of sound during the act of breathing, was proposed. A model of the alveolar wall in the form of a membrane was developed. It is shown that under its periodically stretched, transverse oscillations capable to generate sound can occur. The characteristics of a complex sound signal, which is formed when membranes with different mechanical characteristics are simultaneously excited, are determined. Analysis of this signal showed that its shape and fractal properties are quite close to the shape and fractal properties of real vesicular sound.

KEY WORDS: fractal, multifractal, breathing sound, vesicular sound, tracheal sound.

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P-ANALYTIC FUNCTIONS: WHAT IS IT.

According to the materials of the monograph of Heorhii Mykolaiovych Polozhii "Theory and application of p-analytic and (p,q)-analytic functions"

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This report is devoted to consideration of the definition of p-analitic functions and their main properties. The theory of p-analitic and (p,q)-analitc function was initiated by Heorhii Mykolaiovych Polozhii.

To understand the difference between the analitic and the p-analitic functions let‘s mention the definition of analitic function. The function $f = f(z)$ is an analitic function of complex variable z in complex plane domain $D \subset \mathbb{C}$ if at each point $z \in D$ there is such a limit

$$\lim_{\Delta z \to 0} \frac{f(z + \Delta z) - f(z)}{\Delta z} =: f'(z).$$

If we write the function $f = f(z)$ of complex variable $z = x + iy$ in the form $f(z) = u(x, y) + iv(x, y)$, where u, v are real functions of two real variables x, y, we have the theorem.

Theorem. *The function $f(z) = u(x, y) + iv(x, y)$, $z = x + iy$, is an analitic function of complex variable in domain D if and only if*

1) $u, v \in C^1(D)$ as functions of two real variables x, y;

2) Cauchy-Rieman conditions are met $\quad u'_x = v'_y, \quad u'_y = -v'_x.$

To define the p-analitic function we need to change the Cauchy-Rieman conditions and then we fave the definition:

The function $f(z) = u(x, y) + iv(x, y)$, $z = x + iy$, is a p-analitic function of complex variable in domain $D \subset \mathbb{C}$ when

1) $u, v \in C^1(D)$ as functions of two real variables x, y;

2) the conditions are met $\quad p(x, y)\, u'_x = v'_y, \quad p(x, y)\, u'_y = -v'_x,$

where $p = p(x, y) > 0$ in domain D.

There is the theorem that makes possible to define the p-analitic function define in terms of a limit.

Theorem. *The function $f = f(z) = u(x, y) + iv(x, y)$, $z = x + iy$, of complex variable is a p-analitic function in domane $D \subset \mathbb{C}$ if at each point $z \in D$ there is such a limit*

$$\lim_{\Delta z \to 0} \frac{p\Delta u + \Delta v}{\Delta z} =: \frac{d_p f}{dz}.$$

In Polozhii‘s monograph the complete theory of p-analitic and (p,q)-analitic functions is built. This theory is too close to the theory of clasical analitic functions of a complex variable. But not all theorems from classical theory have analogs for p-analitic and (p,q)-analitic functions.

Also p-analitic and (p,q)-analitic functions have a lot of aplications in differet areas of the mathematical physics, the theory of elastisity. For example, we can solve the axisymmetric problems of the potential theory and the theory of elastisity in domanes with internal cracks and Inhomogeneous inclusions using x-analitic function.

Approximate averaged bounded synthesis for a parabolic process with two switching points: theoretical prove and computational experiment

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The problem of the optimal bounded control with semi-definite quality criterion for parabolic process with rapidly oscillating coefficients on finite time interval is analyzed. It continues the works presented in [1] in which an optimal control problem with a semi-defined quality criterion for a process described by a boundary value problem for a parabolic equation with rapidly oscillating coefficients is investigated. The case when optimal control goes out both on lower, and on upper restriction, that is has two switch-points is considered. Our aim is to build a synthesized control which is practically preferable to program one. But after Fourier method application such a control will contain coefficients which are expressed by series and, in addition, it irregularly depends on small parameter ε. Thus, for practical application it is natural to restrict infinite series by finite sums and to get rid of dependence on a small parameter using average procedure. So, besides the law of optimal synthesis also approximate averaged feedback control, which provides control system behavior that is close to optimal one and thus has a series of advantages from the practical application point of view, is proposed and proved. The efficiency of approximate averaged control construction procedure consisted of cutting to finite sums of series in optimal control and replacing of Fourier coefficients nonregular depended on small parameter by corresponding average values is illustrated by concreate example of controlled system for parabolic process. The example of a specific controlled system for a parabolic process illustrates the effectiveness of the procedure for constructing an approximate averaged control which consists in "cutting"the infinite series included in the expressions for optimal control to finite sums and replacing the Fourier coefficients which irregularly depend on a small parameter with the corresponding averaged values. We compare the properties of the averaged control and the sequence of optimal controls calculated at the different values of the small parameter. For comparison, we use the switching points of optimal control and averaged control, deviation between optimal control and averaged control, the difference in the values of the quality criterion on optimal control and averaged control. In considered example the precision of quality criteria value on approximate control has ε-order and for sufficiently small ε the precision is one order better then ε.

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Multifractal Properties of Human Breathing Noise

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Multifractal analysis of human breathing noise allows to assess the fine (local) structure of these signals. As you know, the main purpose of the parenchyma is to ensure the transition of oxygen from the air into the blood, and carbon dioxide in the opposite direction. This process takes place in the alveoli, the number of which in both lungs of an adult reaches approximately 600 million. The shape of the alveoli resembles an irregular polyhedron of arbitrary orientation. The average transverse size of the alveoli is about 250-300 ?m. There are currently no statistical data on the number of faces (walls of the alveoli), their size and shape. The thickness of the walls of the alveoli is about 2-8 microns. Their total area on exhalation is approximately 30 m2, and on deep inhalation - 100 m2. Thus, it is obvious that the lung parenchyma is a biological material with a porous structure, in which the shape of cells (alveoli) and their orientation in space are disordered. A material with such characteristics can be classified as a fractal material. If we accept the hypothesis that the noise of the parenchyma, which is generated during its stretching-compression deformations, is caused by the oscillations of a huge number of alveolar walls of various shapes and sizes, then the fractal structure of the parenchyma should be manifested in the structure of the noise generated by it. In view of the stated considerations, the report is devoted to the description of the results of the study of the presence of multifractal properties of breathing noise. As it turned out, such studies serve as material to confirm the hypothesis that vesicular noise is formed due to the stretching-compression of the lung parenchyma.

KEY WORDS: fractal, multifractal, breathing sound, vesicular sound, tracheal sound.

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Invariant sets of stochastic differential equations

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Consider a system of stochastic differential equations

$$d\xi(t) = a(\xi(t)dt + b(\xi(t)dw(t), \quad \xi(0) = x^0, \tag{1}$$

where $a(x) = (a_1(x), a_2(x))$, $b(x) = (b_1(x), b_2(x))$ - continuous-differential functions in a certain open domain $D \subset R^2$. Denote by $w(t)$ the one-dimensional Wiener process defined in probabilistic space (Ω, F, P), $x = (x_1, x_2)$ - point in D, $x^0 \in D$. Denote by $\Gamma_D(G)$ the set of the form $\Gamma = \{x : G(x) = C\} \subset D$, where C is a definite constant, $G(x)$ - twice a continuous-differential function in D and has no special points for all $x \in \Gamma$.

If for all $x^0 \in \Gamma_D(G)$ $\quad P\{sup_{0 \leqslant t \leqslant \tau_{D(x^0)}} |G(\xi(t)) - G(x^0)| = 0\} = 1$,

where $\tau_{D(x^0)}$ is the moment of the first exit of the solution from the domain D, then the curve $\Gamma_{D(G)}$ is a locally invariant curve of the corresponding equation (1) .

Consider the problem of investigating the conditions under which the locally phase trajectories of a deterministic differential equation can be locally invariant curves of the corresponding perturbed equation by a random Wiener process of the "white noise"type in the Ito form.

According to [2], the locally invariant curve $\Gamma_{D(G)}$ of equation (1) coincides with the locally phase trajectory of equation

$$dx(t)/dt = b(x(t)), \quad x(0) = x^0. \tag{2}$$

Theorem 1. *Locally phase trajectory $\Gamma_{D(G)}$ of equation (2), in which $|b(x)| > 0$ for all $x \in \Gamma_{D(G)}$ there can be a local phase curve of equation (1), only when the random perturbation of equation (2) by Ito-shaped "white noise"processes occurs along the phase velocity vector of equation (2).*

Theorem 2. *The locally phase trajectory $\Gamma_D(G)$ of equation (2) can be a locally invariant curve of equation (1) in which $(\nabla G(x), a(x)) = 0$ for all $x \in \Gamma_D(G)$, only when the curve consists only of equilibrium points of equation (2) ($|b(x)| = 0$), and points where the curvature of the curve $\Gamma_D(G)$ is zero.*

Theorem 3. *Let the curves $\Gamma_D(G)$ be the set of locally phase trajectories of equation (2) for all C. If the curvature of the curve $\Gamma_D(G)$ is not equal to zero at the point $x^0 \in D$, $|b(x^0)| > 0$ and $(\nabla G(x), a(x)) = 0$ for all $x \in D$, then the solution of equation (2) instantly deviates from $\Gamma_D(G)$ the direction of convexity of the curve at the point x^0.*

The considered results of the research allow building other control models for a disturbed system using the selection of a non-random component.

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Topological structure of functions with isolated critical points on 3-manifolds

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To each isolated critical point of a smooth function on a 3-manifold we put in the correspondence a tree (graph without cycles). We prove that functions are topologically equivalent in some neighborhoods of critical points if and only if the corresponding trees are isomorphic. We investigate critical point graph as topological invariant of an isolated critical point of a smooth function on a 3-manifold. The distinguishing graph, which is a complete topological invariant of functions with three critical points, on a closed 3-manifold, was constructed. It specifies the partition of a closed 3-manifold into three three-dimensional disks. Criteria of topological equivalence and realization theorem was proved. The list of all possible distinguishing graphs of the complexity no more 4 is given.

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Manifestations of nonlinear effects in the angular movement of the tank-liquid system. Examples of system behavior for different cases of excitation

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The problem of the combined motion of a cylindrical tank, partially filled with an ideal liquid, on a pendulum suspension is considered. The results of linear and nonlinear modeling of the behavior of the system under harmonic excitation of a tank on a pendulum suspension in the vicinity of the first resonant frequency and at non-resonant frequencies; under the action of a short-term instantaneous impulse applied to the walls of the tank are presented.

It was ascertained that manifestation of internal mechanisms of nonlinear interaction between normal modes of oscillations of a liquid differs considerably from the case of harmonic moment disturbance on the resonant frequency. The conducted analysis showed the specificity of the formation of non-linear processes of interaction between the normal modes of the free surface, which for various reasons is absent for the cases of external excitation of non-resonant frequencies and transient motion of the system.

It is shown that for the nonlinear case, due to the development of oscillations not only in antisymmetric modes (as in the linear case), but also in the axisymmetric modes and the nodes corresponding to circular number 2 (which are excited through the so-called radiation mechanism), the contribution of the first antisymmetric mode is no longer dominant. It was found that the modes involved by the nonlinear mechanism determine the asymmetry of the wave profiles on the free surface of the liquid, and for the accepted type of harmonic excitation near the first resonance frequency, they mutually cancel each other on the walls of the tank. A similar effect is manifested on short and medium lengths of the pendulum suspension.

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Approximate Stabilization for the One Nonlinear Parabolic Boundary Value Problem

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We consider the optimal stabilization problem: to find a control

$$u(\cdot) \in U = L^2(0, +\infty), \tag{1}$$

which supplies minimal value to functional

$$J(u) = \int_0^{+\infty} \left(\int_\Omega (y^\varepsilon(x,t))^2 dx + \gamma u^2(t) \right) dt, \ \gamma > 0, \tag{2}$$

defined on the solutions of the problem

$$\begin{cases} y_t^\varepsilon(x,t) = \Delta y^\varepsilon(x,t) - \varepsilon f(y^\varepsilon(x,t)) + g(x)u(t), \ x \in \Omega, \ t > 0, \\ y^\varepsilon(x,t) = 0, \ x \in \partial\Omega, \ t > 0, \\ y^\varepsilon(x,0) = \varphi^\varepsilon(x), \ x \in \Omega, \end{cases} \tag{3}$$

where $\Omega \subset \mathbb{R}^n$ is bounded domain, $\varepsilon > 0$ is a small parameter, $g \in L^2(\Omega)$ and nonlinear perturbation $f \in C(\mathbb{R})$ satisfies conditions:

$$\exists C > 0 \ \exists \alpha > 0 \ \exists p \geqslant 2 \ \forall y \in \mathbb{R} \quad |f(y)| \leqslant C(1 + |y|^{p-1}), \quad y \cdot f(y) \geqslant \alpha(y)^p. \tag{4}$$

From [1] we obtain that under conditions (4) for $\forall \varepsilon > 0$, $\varphi^\varepsilon \in L^2(\Omega)$, $u(\cdot) \in U$ there exists at least one (maybe nonunique) solution of (3) in class

$$W = L^2_{loc}(0, +\infty; H^1_0(\Omega)) \cap L^p_{loc}(0, +\infty; L^p(\Omega)) \cap \mathbb{C}([0, +\infty); L^p(\Omega)).$$

Let $\{\tilde{y}^\varepsilon, \tilde{u}^\varepsilon\}$ be an optimal process in (1) – (3), $\widetilde{J}_\varepsilon$ be a corresponding value of the functional. Assume that $\varepsilon = 0$. Then (1) – (3) is the linear-quadratic problem without constraints which under [2] has the unique optimal regulator $u[y]$, stabilizing the solutions of the problem (3).

We consider the problem

$$\begin{cases} y_t(x,t) = \Delta y - \varepsilon f(y) + g(x)u[y], \ x \in \Omega, \ t > 0, \\ y|_{\partial\Omega} = 0, \\ y(0) = \varphi^\varepsilon. \end{cases} \tag{6}$$

Let $\widehat{y}^\varepsilon$ be a solution of the problem (6), $\widehat{u}^\varepsilon := u[\widehat{y}^\varepsilon]$, $\widehat{J}_\varepsilon := J(u[\widehat{y}^\varepsilon])$.

The aim of our work is to substantiate the boundary equality

$$\lim_{\varepsilon \to 0} |\widetilde{J}_\varepsilon - \widehat{J}_\varepsilon| = 0.$$

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Incidence and reflection of Coriolis-dispersed harmonic waves at the boundary of an elastic half-space

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In the modern inertial sensors, based on the bulk acoustic waves propagation, the change of the polarization vector through a certain finite length of the waveguide, is proposed to determine the angular velocity [1,2]. This phenomenon is caused by Coriolis dispersion due to the action of the inertial forces of rotational motion [3,4]. Some issues of calculation of reflected and refracted waves in rotational motion are important for the engineering, these are the joined waveguide, the reflection from a non-orthogonal boundary, etc. So, the problem of the incidence and reflection of Coriolis-dispersed harmonic waves on the surface of an elastic half-space at an arbitrary angle of propagation plays here the key role.

Adhering to the approach proposed in [5], we formulate equations of motion in a relative reference frame. The main variables here are the relative elastic displacements $\mathbf{u}$ and the angular velocity of rotation $\mathbf{\Omega}$ of the moving reference frame. Lamé equation takes a form

$$(c_1^2 - c_2^2)\nabla(\nabla \cdot \mathbf{u}) + c_2^2\nabla^2\mathbf{u} = \ddot{\mathbf{u}} + \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{u}) + 2(\mathbf{\Omega} \times \dot{\mathbf{u}}), \tag{1}$$

where $c_1 = \sqrt{(2G/\rho)(1-\nu)/(1-2\nu)}$ and $c_2 = \sqrt{G/\rho}$ are the wavespeeds of longitudinal and transverse waves in the non-moving elastic solid.

The general solutions for incident and reflected waves are built on the basis of fundamental solutions for quasi-longitudinal and quasi-transverse waves, propagating in the unbounded rotating medium [4]. So the incident and the reflected waves are formed by two coupled components of displacements, that is, such waves have elliptical polarization.

It was found that the laws of Coriolis dispersion do not depend on the angles of incidence or reflection of waves, and the dispersion relations are the same as in an unbounded rotating medium [4]. In the case of a free surface, the angles of incidence and reflection of normal dispersive waves are equal to each other, but the angle of reflection of a tangential dispersive wave is determined by the ratio of dispersive wavenumbers. In the case of small angular velocity Ω compared to the wave frequency ω one can suggest such asymptotic eavaluations for the reflection coefficient κ and for the angle of reflection ϑ_2

$$\kappa = \frac{c_1}{c_2}\left(1 + 2\frac{\Omega^2}{\omega^2}\frac{c_1^2 + c_2^2}{c_1^2 - c_2^2}\right), \qquad \vartheta_2 = \arcsin\left[\frac{c_1}{c_2}\left(1 + 2\frac{\Omega^2}{\omega^2}\frac{c_1^2 + c_2^2}{c_1^2 - c_2^2}\right)\sin\vartheta_0\right]. \tag{2}$$

Here ϑ_0 is the angle of incidence of dispersive wave.

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ABSTRACTS

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