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ABSTRACTS

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LOCALLY MATRIX ALGEBRAS

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Let F be a ground field. An associative F-algebra A is called a locally matrix algebra if for each finite subset of A there exists a subalgebra containing this finite subset which is isomorphic to the matrix algebra $M_n(F)$ for some n.

We will start with a survey of abstract theory of locally matrix algebras, and we will discuss the tensor decompositions of locally matrix algebras and their Steinitz invariants.

Special attention will be paid to the properties of their automorphisms groups and their Lie algebras of derivations. We also describe automorphisms and derivations of Mackey algebras and Mackey groups. In particular, we describe automorphisms of all infinite simple finitary torsion groups (in the classification of J.Hall) and derivations of all infinite-dimensional simple finitary Lie algebras (in the classification of A.Baranov and H.Strade).

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LONG-TIME BEHAVIOR OF A STOCHASTIC MODELS OF POPULATION DYNAMICS WITH JUMPS

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In this report, we consider the non-autonomous stochastic models of population dynamics driven by stochastic differential equations or by the system of stochastic differential equations with white noise, centered and non-centered Poisson noises. So, we take into account not only "small" jumps, corresponding to the centered Poisson measure, but also the "large" jumps, corresponding to the non-centered Poisson measure. The coefficients of corresponding stochastic logistic differential equations do not satisfy the linear growth condition but satisfy the local Lipschitz condition. Therefore, there exists a local solution to the corresponding stochastic differential equations. In the cases of the mutualism model and predator-prey model the coefficients of corresponding stochastic differential equations satisfies neither the local Lipschitz condition nor the linear growth condition. The solution of corresponding stochastic differential equations must be positive because they represent the size of the population. We presented the theorems on the existence and uniqueness of a global, positive solution to the corresponding stochastic differential equation and system of stochastic differential equations.

For considered models we derived sufficient conditions of stochastic permanence, non-persistence in the mean, weak and strong persistence in the mean and extinction of the populations.

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ON SOME CONDITIONS FOR SECOND MODULI OF CONTINUITY

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Let $C_{2\pi}$ be the class of all continuous 2π-periodical real functions, which are defined on $\mathbb{R}$, $UC(\mathbb{R})$ – the class of all uniformly continuous real functions which are defined on $\mathbb{R}$.

Second module of continuity for a function $f : \mathbb{R} \to \mathbb{R}$ is the function

$$\omega_2(f,h) = \sup_{t\in[0,h]} \sup_{x\in\mathbb{R}} |\triangle_t^2 f(x)|, \ h \geqslant 0,$$

The problem of a simple characterization of all functions which are second moduli of continuity for functions that belong to one the mentioned classes is open and appeared to be hard to solve. Some characterizations up to order equivalence were obtained in the works of I. Shevchuk and V.Geit.

So the problem of obtaining of necessary or sufficient conditions on function to guarantee that it is a second module of continuity is interesting and actual. V. Geit obtained one such sufficient condition:

Theorem (V.Geit) *A function $\varphi(t), \ 0 \leqslant t \leqslant \pi$ is the second module of continuity for a function from the class $C_{2\pi}$, if:*
a) $\varphi(0) = 0$;
b) φ – nondecreasing and continuous on $[0,\pi]$;
c) even and 2π–periodical continuation $\hat{\varphi}$ of the function φ fulfill the condition

$$|\triangle_t^2 \hat{\varphi}(x)| \leqslant 2\varphi(|t|), \ x\in\mathbb{R}, \ |t| \leqslant \pi.$$

Geit mentioned that the question about correctness of inverse statement was open. Here are answers for that question for both mentioned spaces.

Theorem 1. *There are a function φ, which is the second module of continuity for some function from the class $C_{2\pi}$, such that even 2π-periodic continuation $\hat{\varphi}$ of the function φ from the segment $[0,\pi]$ for some $x_0 > 0, \ t_0 > 0$ fulfill the condition*

$$\triangle_{t_0}^2 \hat{\varphi}(x_0) > 2\varphi(|t_0|).$$

Theorem 2. *There are a function φ, which is the second module of continuity for some function from the class $UC(\mathbb{R})$, such that the even continuation $\hat{\varphi}$ of the function φ for some $x_0 > 0, \ t_0 > 0$ fulfill the condition*

$$\triangle_{t_0}^2 \hat{\varphi}(x_0) > 2\varphi(|t_0|).$$

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EVOLUTIONARY EQUATIONS FOR THE SYSTEM OF HARD SPHERES

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This talk discusses with the mathematical problems of the description of the evolution of many hard spheres based on various ways of describing their state, namely by means of functions describing the propagation of correlations. One of the developed approaches allows one to describe the evolution of both a finite and an infinite average number of hard spheres using reduced distribution functions or reduced correlation functions, which are determined by the dynamics of correlations of a hard-sphere system [1], [2]. We note the importance of the description of the processes of the creation and propagation of correlations, in particular, it is related to the problem of the description of the memory effects in many-particle systems with collision dynamics.

It was established that the notion of cumulants of the groups of operators underlies nonperturbative expansions of solutions for the fundamental evolution equations describing the state evolution of a hard-sphere system, namely, of the Liouville hierarchy for correlation functions, of the BBGKY hierarchy for reduced distribution functions and of the nonlinear BBGKY hierarchy for reduced correlation functions, as well as it underlies the kinetic description of infinitely many hard spheres [1], [2]. We emphasize that the structure of obtained expansions for correlation functions, in which the generating operators are the cumulants of the corresponding order of the groups of operators of hard spheres, induces the cumulant structure of series expansions for reduced distribution functions, reduced correlation functions and reduced correlation functionals. Thus, the dynamics of systems of infinitely many hard spheres is generated by the dynamics of correlations.

The origin of the collective behavior of a hard-sphere system on a microscopic scale was described by means of a one-particle correlation function that is determined by the non-Markovian Enskog kinetic equation. The advantages of such an approach to the derivation of kinetic equations from underlying collisional dynamics consists of an opportunity to construct the kinetic equations with initial correlations, which makes it possible to describe the propagation of initial correlations in the Boltzmann-Grad limit [3]. Another advantage of this approach is related to the rigorous derivation of the Boltzmann equation with higher-order corrections to the main term of the Boltzmann-Grad asymptotics of collisional dynamics.

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The coupling method application to the study of stability of time-inhomogeneous Markov chains

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The coupling method is a powerful tool for studying the stability of homogeneous Markov chains. The typical stability problem requires estimating a norm

$$||P(X_n \in \cdot | X_0 = x) - P(X_n \in \cdot | X_0 = y)||$$

which is a difference of n-steps transition probabilities for the chain starting with different initial points. We modified the method to study the stability of two different time-inhomogeneous processes.

$$||P(X_n \in \cdot | X_0 = x) - P(X'_n \in \cdot | X'_0 = y)||.$$

Estimating such a norm is important for both $x = y$ and $x \neq y$.

The key tools in establishing an estimate for such a norm are renewal theory and recurrent moment estimation. For example, the existence of the exponential moment $E_x[\beta^{\sigma_C}]$, where $\beta > 1$ some constant and σ_C is the first return time to the set C is enough to prove that n-steps transition probabilities remain close even for two different chains provided a proximity of one-steps probabilities.

The following condition is a generalized classical Foster-Lyapunov criterion, and we proved that it guarantees the existence of such moment to a time-inhomogeneous chain.

We say that a sequence of Markov kernels $(P_t)_{t \geqslant 0}$ satisfies **Condition (D)** with the set C if:

1. There exists a sequence of positive integers $\{n_k, k \geqslant 1\}$, a sequence of measurable functions $V_k \colon E \to [1, \infty]$ and two sequences of positive constants $\{\lambda_k,\ k \geqslant 0\}$, and $\{b_k,\ k \geqslant 0\}$ such that for all $x \in E$

$$P^{N_k, n_{k+1}} V_{k+1}(x) \leqslant \lambda_{k+1} V_k(x) + b_k \mathbb{1}_C(x), \tag{1}$$

where $N_k = \sum\limits_{j=1}^{k} n_j,\ k \geqslant 1$.

2. Sequence $\{\lambda_k,\ k \geqslant 0\}$ defined in item 1., satisfies

$$\sum_{k=0}^{\infty} \left(\prod_{j=0}^{k} \lambda_j \vee 1 \right)^{-1} (1 - \lambda_k)^+ = \infty.$$

Here $a \vee b = \max\{a, b\}$, and $a^+ = \max\{a, 0\}$.

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On monogenic inverse semigroup and semigroup of partial automorphism of a rooted tree

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An inverse semigroup is a semigroup S with the property that for every element $a \in S$ there is a unique $b \in S$ such that

$$aba = a \text{ and } bab = b.$$

Let A be a nonempty alphabet. We extend

$$^{-1} : A \to A^{-1} : \quad a \mapsto a^{-1}$$

$$(a^{-1})^{-1} = a, \quad (ua)^{-1} = a^{-1}u^{-1} \quad (u \in A^*;\ a \in A).$$

The free inverse semigroup on A is the quotient

$$FIS(A) = A^*/\nu,$$

where ν is the congruence on A^* generated by the relation

$$\{(ww^{-1}w, w) | w \in A^*\} \cup \{(uu^{-1}vv^{-1}, vv^{-1}uu^{-1}) | u, v \in A^*\}.$$

If $A = \{a\}$, then we have free inverse semigroup.

Let Ω be a non-empty set. We consider the set of all partial one-to-one mappings. It forms an inverse semigroup under natural composition law. Denote by $IS(\Omega)$ the inverse semigroup of all partial one-to-one mappings on Ω.

A partial one-to-one mapping $f \in IS(\Omega)$ is called:

- a finite cycle of length $k \geqslant 1$, if $\operatorname{dom} f = \{\omega_1, \ldots, \omega_k\}$ and
$$f(\omega_1) = \omega_2, \ldots, f(\omega_{k-1}) = \omega_k, f(\omega_k) = \omega_1;$$
- an infinite cycle, if $\operatorname{dom} f = \{\omega_i : i \in \mathbb{Z}\}$ and
$$f(\omega_i) = \omega_{i+1}, \quad i \in \mathbb{Z};$$
- a finite chain of length $k \geqslant 0$, if $\operatorname{dom} f = \{\omega_1, \ldots, \omega_k\}$ and for some ω_{k+1}
$$f(\omega_1) = \omega_2, \ldots, f(\omega_k) = \omega_{k+1}, f(\omega_{k+1}) = \varnothing;$$
- an infinite left chain, if $\operatorname{dom} f = \{\omega_i : i \in \mathbb{N}\}$ and
$$f(\omega_1) = \varnothing, \quad f(\omega_i) = \omega_{i-1}, \quad i \geqslant 2;$$
- an infinite right chain, if $\operatorname{dom} f = \{\omega_i : i \in \mathbb{N}\}$ and
$$f(\omega_i) = \omega_{i+1}, \quad i \geqslant 1.$$

Every partial mapping f from $IS(\Omega)$ can be is uniquely represented as a product of disjoint finite or infinite cycles, and/or a finite or infinite chains.

Theorem. *Let Ω be an infinite set, $f \in IS(\Omega)$. The following statements are equivalent:*

1. *the inverse subsemigroup generated by f is monogenic free inverse;*
2. *for arbitrary $n \geqslant 1$ there exist $\omega_1, \omega_2 \in \Omega$ such that $f^n(\omega_1) \in \operatorname{dom} f, f^{-n}(\omega_2) \in \operatorname{ran} f$, but $f^{n+1}(\omega_1) \notin \operatorname{dom} f, f^{-(n+1)}(\omega_2) \notin \operatorname{ran} f$;*
3. *the cycle–chain decomposition of f contains infinite left and right chains or for arbitrary $n \geqslant 1$ a finite chain of length greater than n.*

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ONE GENERALIZATION OF CAUSHY THEOREM TO THE CASE OF DIFFERENTIABLE FUNCTIONS OF SEVERAL PARAMETERS

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Let $t = (t_1, t_2, \ldots, t_m)\,, h = (h_1, h_2, \ldots, h_m)\,, h_i > 0, 1 \leqslant i \leqslant m$; $\Pi = [t_1, t_1 + h_1] \times \ldots \times [t_m, t_m + h_m]$ be the m–dimensional parallelepiped in the R^m; $f : \Pi \to R$ be the real function of the m real variables.

We define the increment of a function f on a parallelepiped Π by the equality

$$\Delta_\Pi f = (-1)^m \sum_{\alpha_1=0}^{1} \ldots \sum_{\alpha_m=0}^{1} (-1)^{\alpha_1+\ldots+\alpha_m} f\left(t_1 + \alpha_1 h_1, \ldots, t_m + \alpha_m h_m\right).$$

Theorem 1. *Let* $\Pi = [a_1, b_1] \times [a_2, b_2]\,[a_1, b_1] \ldots [a_m, b_m]$ *be the parallelepiped in the Euclidean space* R^m*; the functions* $f, g : \Pi \to R$ *satisfy these conditions:*

1. $f, g \in C^{(m)}(\Pi)$;
2. $\forall x = (x_1, x_2, \ldots, x_m) \in \Pi : \frac{\partial^m g(x_1,x_2,\ldots,x_m)}{\partial x_1 \partial x_2 \ldots \partial x_m} \neq 0$.

Then there is a point $\xi = (\xi_1, \xi_2, \ldots, \xi_m) \in \Pi$*, that*

$$\frac{\Delta_\Pi f}{\Delta_\Pi g} = \frac{\frac{\partial^m f(\xi_1,\xi_2,\ldots,\xi_m)}{\partial x_1 \partial x_2 \ldots \partial x_m}}{\frac{\partial^m g(\xi_1,\xi_2,\ldots,\xi_m)}{\partial x_1 \partial x_2 \ldots \partial x_m}}.$$

This theorem generalizes Caushy theorem for differentiable functions to the case of m times continuously differentiable functions of the m variables.

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On the asymptotic analysis of unstable solutions of stochastic differential equations in the creative scientific works of G. L. Kulinich

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Let ξ be the solution of the stochastic differential equation

$$\xi(t) = x_0 + \int_0^t a\left(\xi(s)\right)\, ds + W(t), \quad t \geqslant 0, \tag{1}$$

where $W = \{W(t), t \geqslant 0\}$ is a Wiener process, $a = a(x)$ is a continuous, absolutely integrable on the whole axis function with

$$\int_{\mathbb{R}} a(x)\, dx = \lambda. \tag{2}$$

The problem (1) – (2) can be considered as the description of an external perturbation by the coefficient a of the physical environment described by the Wiener process. In particular, under the condition $\lambda \neq 0$ we can assume that at the initial point $x = 0$ an energy source of a high power is implemented in a homogeneous environment. The main motivation for considering of our model (1) and (2) is the fact that it can be used in the mathematical description of anomalous phenomena. This is, for example, underwater volcanic eruptions or nuclear explosions, tsunamis, tornadoes and other turbulence, while in the classical case, the motion of a Brownian particle in "smooth" media is considered.

A new asymptotic method of the investigation such a model was proposed by G. L. Kulinich and the first results in this direction were obtained in the paper [1]. Similar problems for unstable solutions of systems of stochastic differential equations are considered, for example, in the papers [2], [3] and [4]. On the earliest stages of his research, G. L. Kulinich was inspired and directed by his teacher A. V. Skorokhod, whose ideas influenced the overall content of the book [5].

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WIENER-HOPF SYSTEMS IN THE PROBLEMS OF THE PLANE THEORY OF ELASTICITY

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In this report we consider the Wiener-Hopf system of special form, that is used to solve the problem of the theory of elasticity if the half-plane $y > 0$ with the cut lying at an angle to the fixed boundary $y = 0$.

If we separate the half-plan along the straight half-line, that include the cut, we obtain two sub-domains $\Omega_1 = \{\rho > 0, 0 < \varphi < \alpha\}$ and $\Omega_2 = \{\rho > 0, \alpha < \varphi < \pi\}$, $0 < \alpha \leqslant \pi/2$ is the cut slope angle, ρ, φ are the polar coordinates. Next we need to find the solutions of the Lame equations in each sub-domain and these solutions must satisfy boundary conditions for the displacement vector components on the boundary $y = 0$, boundary conditions for the stress tensor components on the cut and conjugate conditions on the cut prolongation. The solutions are built in the Papkovich-Neiber form using the Melline integral transform. After fulfilling all the conditions the problem is reduced to the system of Wiener-Hopf functional equations in a complex plane

$$\vec{Y}_-(p) - \frac{1}{p+1}\vec{P}_0 = -\frac{p}{\sin(\pi p)}\mathbf{M}(p)\vec{X}_+(p), \quad |\mathrm{Re}\, p| < \varepsilon \leqslant 1, \tag{1}$$

where $\vec{Y}_-(p) \in \mathsf{A}\left(\mathrm{Re}\, p < \varepsilon\right), \vec{X}_+(p) \in \mathsf{A}\left(\mathrm{Re}\, p > -\varepsilon\right)$ are unknown vector-functions with known behavior, when $p \to \infty$; $\vec{P}_0$ is a known vector, that describes the stresses' distributions on the cut.

The matrix $\mathbf{M}(p)$ has the special form and can be written in so called "trigonometric form"

$$\mathbf{M}(p) = a(p)\mathbf{E} + b(p)\mathbf{J}(p) = \sqrt{\det \mathbf{M}(p)}\ \left(\cos\theta(p)\cdot\mathbf{E} + \frac{\sin\theta(p)}{\sqrt{\delta(p)}}\cdot\mathbf{J}(p)\right),$$

where $a, b \in \mathsf{A}\left(|\mathrm{Re}\, p| < \varepsilon\right)$ are known functions, $\mathbf{E}$ is the unit matrix of 2x2 size, $\mathbf{J}(p)$ is a matrix with polynomial elements and $\mathbf{J}^2(p) = -\delta(p)\mathbf{E}$, $\delta(p)$ is the even polynomial of the 2nd degree, $\delta(p) \neq 0$, when $|\mathrm{Re}\, p| < \varepsilon$, $\det\mathbf{M}(p) = a^2(p) + b^2(p)\delta(p) \neq 0$, if $|\mathrm{Re}\, p| < \varepsilon$, $\theta(p) = \frac{1}{2i}\ln\frac{a(p)+ib(p)\sqrt{\delta(p)}}{a(p)-ib(p)\sqrt{\delta(p)}} \in \mathsf{A}\left(|\mathrm{Re}\, p| < \varepsilon\right)$.

The main step of the system (1) solving is the matrix $\mathbf{M}$ factorization

$$\mathbf{M}(p) = \mathbf{M}_-(p)\cdot\mathbf{M}_+(p), \quad |\mathrm{Re}\, p| < \varepsilon,$$

where $\mathbf{M}_-(p) \in \mathsf{A}\left(\mathrm{Re}\, p < \varepsilon\right)$ and $\mathbf{M}_+(p) \in \mathsf{A}\left(\mathrm{Re}\, p > -\varepsilon\right)$. The matrix factorization is more complex problem, then a function one. But in this case we can represent matrices $\mathbf{M}_\pm(p)$ in the same trigonometric form. Then the matrix $\mathbf{M}$ factorization problem transforms into the factorization problem for functions $\sqrt{\det\mathbf{M}}$ and θ. It makes possible to solve the Wiener-Hopf system (1) in complete form.

The found solution of the system (1) gives us the complete solution of the problem of the theory of elasticity and makes possible to describe the stress distributions near the cut, to find the singularity at the end of the cut, that lies on the half-plane bound.

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Regression analysis of mixture with varying concentrations

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We continue studies of the jackknife (JK) technique application for estimation of estimators' covariance matrices in models of mixture with varying concentrations (MVC) [2, 3]. On JK applications for homogeneous samples, see [1]. In MVC models one deals with a nonhomogeneous sample, which consists of subjects belonging to different sub-populations (mixture components). One knows the probabilities with which a subject belongs to the mixture components and these probabilities are different for different subjects. Therefore, the considered observations are independent but not identically distributed.

We consider objects from a mixture with various concentrations. All objects from the sample Ξ_n belongs to one of M different mixture components. Each object from the sample $\Xi_n = (\xi_i)_{i=1}^n$ has observed characteristics $\xi_i = (X_i, Y_i) \in R^D$ and one hidden κ_i. $\kappa_i = m$ if i-th objects belongs to the m-th component. These numbers are unknown, but we know the mixing probabilities $p_{i;n}^m = P(\kappa_i = m)$. The X_i is a vector of regressors and Y_i is a response in the regression model

$$Y_i = g(X_i, b^{(\kappa_j)}) + \varepsilon_i \ (1)$$

Here $b^{(m)} \in \Theta \in R^d$ is a vector of unknown regression parameters for the m-th component, the $g : R^{D-1} \times \Theta \to R$ is a known regression function, ε_i is a regression error term. Random variables X_i, ε_i are independent and their distribution is different for different components. The estimator $b_n^{\hat{(k)}}$ for the regression parameter $b^{(k)}$ is a measurable solution to the GEE equation

$$S_n^k(\gamma) = \sum_{j=1}^{n} a_{j;n}^k (Y_j - g(X_i, \gamma))\dot{g}(X_i, \gamma) = 0 \ (2)$$

This equation might have more than one solution. Any of these solutions could be taken as $b_n^{\hat{(k)}}$ estimator. The symbol $\dot{g}(X_i, \gamma)$ means the gradient of function g by the γ term. $a_{j;n}^k$ are the minimax weights defined in [4]. The minimax weights matrix $A_{;n} = (a_{j;n}^k)_{j=1,k=1}^{n,M}$ defined using the mixing matrix $P_{;n} = (p_{j;n}^k)_{j=1,k=1}^{n,M}$:

$$A_{;n} = (a_{j;n}^k)_{j=1,k=1}^{n,M} = P_{;n}(P_{;n}^T P_{;n})^{-1} \ (3)$$

In [3] it is shown that under suitable conditions $b_n^{\hat{(k)}}$ are asymptotically normal, with asymptotic covariance matrix

$$V^{(k)} = \lim_{n\to\infty} (M^{(k)})^{-1} Var S_n^k(b^{(k)})(M^{(k)})^{-T}; M^{(k)} = -E\dot{S}_n^k(b^{(k)})$$

In [5] it is shown that under sutable conditions the jackknife estimator ${}^{JK}V_n^{(k)}$ is consistent to $V^{(k)}$.

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ON HIGHER ORDER MODULI OF CONTINUITY WHICH ARE GENERATED BY A SEMIGROUP OF OPERATORS

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The report was prepared based on the results obtained jointly with S. I. Bezkryla and A. V. Chaikovskyi and published in works [1] - [3].

Let X is a linear space, $\{T_h \ : \ h \geqslant 0\}$ is a one-parameter family of linear operators $T_h : X \to X,\ h \geqslant 0$, which forms a semigroup, that is $T_0 = I$ is a identity operator and $T_{h_1+h_2} = T_{h_1}T_{h_2}$ for arbitrary $h_1 \geqslant 0$ and $h_2 \geqslant 0$.

Let there be a linear set $Y \subset X$ on which a norm $||\cdot||$ is introduced, relative to which the space Y is Banach space, and for all $f \in X$ and $h \geqslant 0$ there is an inclusion $(T_h - I)f \in Y$ and $||T_hf - f|| \to 0,\ h \to 0$. Then for any $h \geqslant 0$ the operator $T_h : Y \to Y$. Let also for every $h \geqslant 0$ restriction of the operator T_h on Y, which we denote $\tilde{T_h}$ be a continuous operator and its norm $||\tilde{T_h}|| \leqslant 1$.

The modulus of continuity of an element $f \in X$ of order $k \in \mathbb{N}$, generated by a semigroup $\{T_h \ : \ h \geqslant 0\}$, is the function

$$\omega_k(t) := \omega_k(f,t) := \sup_{h\in[0,t]} \left\|(I - T_h)^k f\right\|,\ t \geqslant 0.$$

Lemma. *Let* $f \in X,\ \omega(t) := \omega_k(f,t)$ *is the modulus of continuity of the element* $f \in X$ *of order* $k \in \mathbb{N}$*, which is generated by the semigroup* $\{T_h \ : \ h \geqslant 0\}$*. Then:*
1)$\omega(0) = 0$; 2)$\omega \uparrow$ *on* $[0,+\infty)$; 3)$\omega \in C[0,+\infty)$; 4)$\omega(nt) \leqslant n^k\omega(t)$ *for all* $t \geqslant 0$ *and* $n \in \mathbb{N}$.

For arbitrary functions $\omega : [0,+\infty) \to [0,+\infty)$ condition 4) follows from the condition:

5) the function $\omega(t)/t^k$ does not increase monotonically on $(0,+\infty)$.

k-**majorants** are functions $\omega : [0,+\infty) \to \mathbb{R}$, that satisfy conditions 1) - 3) and 5).

The problem of I. O. Shevchuk. Is it correct that every k-majorant given on some segment $[0,\delta_0],\ \delta_0 > 0$, is a uniform modulus of continuity of the k-th order of some function?

The answer is negative. This was established by:

S. V. Konyagin, 2010: $k = 2, X = UC(\mathbb{R})$ is the space of uniformly continuous on $\mathbb{R}$ functions $f : \mathbb{R} \to \mathbb{R}$ and $(T_hf)(x) := f(x + h),\ x \in \mathbb{R}$;

S. I. Bezkryla, O. N. Nesterenko, A. V. Chaikovskyi, 2012-2016: $k \in \mathbb{N},\ k \geqslant 2, X$ is a linear space, a module of continuity is generated by a semigroup of operators. The key point in the proof is the inequality:

$$2\omega_k(f,nt) \leqslant \omega_k(f,(n+1)t) + \omega_k(f,(n-1)t) + 2(2^k - 1)n^{k-2}\omega_k(f,t),\ t > 0.$$

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EXPONENTIATION, p-AUTOMATA AND HNN-EXTENSIONS OF FREE ABELIAN GROUPS

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Let (G, X) and (H, Y) be permutation groups. The wreath product of the permutation group (G, X) with the abstract group H is defined as

$$G \wr H = \{[g, h(x)] : g \in G, h(x) \in H^X\}.$$

The exponentiation [1] of (H, Y) by (G, X) is the permutation group

$$(G \wr H, Y^X)$$

such that every $[g, h(x)] \in G \wr H$ acts on $f(t) \in Y^X$ by the rule

$$f(t)^{[g,h(x)]} = (f(t^g))^{h(t)}.$$

Theorem. *Let p be a prime, G and H be finite p-groups faithfully acting on sets X and Y of cardinalities p^n and p^m correspondingly, $n, m \geqslant 0$. Then the exponentiation of H by G is isomorphic as a permutation group to the wreath product of $m \cdot p^n$ copies of the regular cyclic group of order p acting by automorphisms on the set of leaves of the p-regular rooted tree of depth $m \cdot p^n$.*

This theorem is applied to show that a wide class of HNN-extensions of free abelian groups can be generated by so-called finite p-automata.

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CENTRALIZERS OF LINEAR AND JACOBIAN DERIVATIONS

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Let $\mathbb{K}$ be an algebraically closed field of characteristic zero, $A := \mathbb{K}[x_1, \ldots, x_n]$ the polynomial ring. Recall that a $\mathbb{K}$-linear map $D : A \to A$ is called a $\mathbb{K}$-derivation of the algebra A if $D(fg) = D(f)g + fD(g)$ for any $f, g \in A$. All $\mathbb{K}$-derivations of the $\mathbb{K}$-algebra A form the Lie algebra $W_n(\mathbb{K})$ over the field $\mathbb{K}$ with respect to the commutation and every element $D \in W_n(\mathbb{K})$ can be uniquely written in the form $D = f_1\partial_1 + \cdots + f_n\partial_n$, where $f_i \in A$ i $\partial_i := \frac{\partial}{\partial x_i}$ are partial derivatives. Note that from the geometrical point of view, $W_n(\mathbb{K})$ is the Lie algebra of all vector fields on $\mathbb{K}^n$ with polynomial coefficients; from the viewpoint of differential equations, with any derivation $D = f_1\partial_1 + \cdots + f_n\partial_n$ one can associate an autonomous system of ordinary differential equations

$$\dot{x} = F(x), \text{ where } x = x(t) = (x_1(t), \ldots, x_n(t)), \; F(x) = (f_1(x), \ldots, f_n(x)).$$

So, the structure of $W_n(\mathbb{K})$ is of great interest. For a given derivation $D \in W_n(\mathbb{K})$ one can consider its centralizer $C_{W_n(\mathbb{K})}(D)$ in $W_n(\mathbb{K})$, i.e the set of all elements of the Lie algebra $W_n(\mathbb{K})$ that commute with D; we will denote it briefly by $C_{W_n}(D)$). The centralizer $C_{W_n}(D)$ is obviously a subalgebra of the Lie algebra $W_n(\mathbb{K})$ and $D \in C_{W_n}(D)$. The problem of describing centralizers of elements $D \in W_n(\mathbb{K})$ is closely related to some problems from theory of differential equations and geometry, it is solved only in some special cases (see for example, [1], [2], [3]). We study centralizers of elements in two important classes of derivations on A, namely for linear and for Jacobian derovations. Recall that a derivation $D \in W_n(\mathbb{K})$ is called a linear derivation if $D = \sum_{i,j=1}^{n} a_{ij}x_j\frac{\partial}{\partial x_i}$, $a_{ij} \in \mathbb{K}$. In case when the matrix (a_{ij}) has the Jordan normal form consisting of a single Jordan block, the centralizer of D in $W_n(\mathbb{K})$ is completely described [1]. A derivation D on the polynomial ring $\mathbb{K}[x, y]$ is called a Jacobian derivation if there exists $f \in \mathbb{K}[x, y]$ such that $D(h) = \det J(f, h)$ for any $h \in \mathbb{K}[x, y]$ (here $J(f, h)$ is the Jacobian matrix for f and h). Such a derivation is denoted by D_f. The kernel of D_f in $\mathbb{K}[x, y]$ is a subalgebra $\mathbb{K}[p]$, where $p = p(x, y)$ is a polynomial of smallest degree such that $f(x, y) = \varphi(p(x, y)$ for some $\varphi(t) \in \mathbb{K}[t]$. Let $C = C_{W_2(\mathbb{K})}(D_f)$ be the centralizer of D_f in $W_2(\mathbb{K})$. We prove that C is the free $\mathbb{K}[p]$-module of rank 1 or 2 over $\mathbb{K}[p]$ and point out a criterion of being the module C of rank 2 [2]. These results are used to obtain a class of integrable autonomous systems of differential equations.

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Parameter estimation for mixed fractional Brownian motion

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We consider the following model of mixed fractional Brownian motion with trend:

$$X_t = \theta t + \sigma W_t + \kappa B_t^H, \quad t \geqslant 0,$$

driven by the Wiener process W and an independent fractional Brownian motion B^H with Hurst index H.

Two approaches for estimation of the four unknown parameters θ, σ, κ, and H from discrete observations of the process X have been developed and compared.

The first algorithm is more traditional: the parameters σ, κ, and H are estimated using realized quadratic variations, while the estimator of the parameter θ is constructed by discretizing the maximum likelihood estimator for the continuous-time model. Strong consistency conditions for all estimators are established. This approach has some limitations, including the assumption that $H < \frac{3}{4}$, and some estimators have a very slow rate of convergence.

Therefore, a new method for simultaneous estimation of all four parameters has been developed, based on the use of the ergodic theorem. The strong consistency and joint asymptotic normality of the proposed estimators are established, with an explicit form of the asymptotic covariance matrix.

The quality of the obtained estimators is illustrated through computer simulations, and a comparison between the two approaches described above is performed.

A more general model is also investigated, which includes a linear combination of two independent fractional Brownian motions with different Hurst indices. Strongly consistent estimators for all parameters of this model are constructed, and their joint asymptotic normality is proved.

The results are published in the series of four papers [1–4].

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EXISTENCE, UNIQUENESS SOLUTION AND INVARIANT MEASURE RESULTS FOR NEUTRAL FSDES IN HILBERT SPACES WITH NON-LIPSCHITZ COEFFICIENTS. CONTROLLABILITY

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We study the large time behaviour of the solutions of neutral type stochastic functional-differential equations of the form

$$d[u(t)+g(u_t)] = [Au+f(u_t)]dt+\sigma(u_t)dW(t)\, t>0;$$

$$u(t)=\varphi(t), t\in[-h,0), h>0.$$

Here A is an inifinitesimal generator of a strong continuous semigroup $S(t), t\geqslant 0$ of bounded linear operators in real separable Hilbert space H. The noise $W(t)$ is a Q-Wiener process on a separable Hilbert space K. For any $h>0$ denote $C_h := C([-h,0],H)$ to be a space of continuous H-valued functions $\varphi:[-h,0]\to H$, equipped with the norm

$$\| \varphi \| := \sup_{t\in[-h,0]} ||\varphi(t)||_H,$$

where $||\cdot||_H$ stands for the norm in H. The functionals f and g map C_h to H, and $\sigma : C_h \to L_0^2$, where $L_0^2 = L(Q^{\frac{1}{2}}K, H)$ is the space of Hilbert-Schmidt operators from $Q^{\frac{1}{2}}K$ to H. In our studies, the maps f and g do not satisfy the Lipschitz condition. Therefore, it is important for applications. Finalli , $\varphi : [-h,0]\times\Omega \to H$ is the initial condition, where (Ω, F, P) is the probability space. We study the existence and uniqueness of the solution to the initial problem without the Lipschitz condition. Then we establish the Markov and Feller properties in the shift spaces for such equations, and using the compactness approach we establish the existence of invariant measures in the shift spaces for such equations. The obtained abstract results are applied to stochastic partial differential equations of the reaction-diffusion type. The issue of approximate control of such equations is also studied.

SUB-GAUSSIAN STORAGE PROCESS

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Consider a queue with a single server that receives a sub-Gaussian random source $X = \{X(t), t \in B\}$, where B is some parametric set, for example, a finite segment $[a, b]$ or $[0, T]$. The object of study is storage process

$$Q(t) = \sup_{s \in T\colon s \leqslant t} \left(X(t) - f(t) - (X(s) - f(s))\right),$$

where $f(t)$ is a continuous function that reflects the intensity of queue service.

Let (B, ρ) be a pseudometric compact space. Put $B_t = \{u \in B\colon u \leqslant t\}$ and let $H_{B_t}(u)$ and $N_{B_t}(u)$ be metric entropy and massiveness on B_t.

Assumption Σ. Random process $X = \{X(t), t \in B\}$ satisfies assumption Σ if there exists such a continuous monotone increasing function $\sigma = \{\sigma(h), h > 0\}$ that $\sigma(h) \to 0$ as $h \to 0$ and $\sup\limits_{\rho(t,s) \leqslant h} \tau(X(t) - X(s)) \leqslant \sigma(h)$, where $\tau(\cdot)$ is the sub-Gaussian standard of X.

Assumption δ. Continuous function $f = \{f(t), t \in B\}$ satisfies assumption δ if $|f(u) - f(v)| \leqslant \delta(\rho(u, v))$, where $\delta = \{\delta(s), s > 0\}$ is a nonnegative monotone increasing function.

The following statement contains the conditions for the boundedness of the storage process $Q(t)$ and and estimate for the exponential moment of this process.

Theorem. *Let $X(t) = \{X(t),\ t \in B\}$ be a separable sub-Gaussian process satisfying assumption Σ, and let $f = \{f(t), t \in B\}$ be a continuous function satisfying assumptionδ. Suppose that $r = \{r(u), u \geqslant 1\}$ is a continuous function such that $r(u) > 0$ as $u > 1$ and $s(t) = r(\exp\{t\}), t \geqslant 0$, is convex. If $\int\limits_0^\beta \frac{r(N_{B_t}(\sigma^{(-1)}(u)))}{(H_{B_t}(\sigma^{(-1)}(u)))^{1/2}} du < \infty$ then for all $p \in (0; 1)$ and $x > 0$ the following inequallity holds true*

$$\boldsymbol{P}\{Q(t) > x\} \leqslant Z_r(p, t, x),$$

where $Z_r(p, t, x) = \inf\limits_{\lambda > 0} W_1(\lambda, t, p) \exp\left\{p/2 \left(\frac{\lambda\beta}{1-p}\right)^2 + \lambda\left(\sum\limits_{k=2}^{\infty} \delta\left(\sigma^{(-1)}\left(\beta p^{k-1}\right)\right) - x\right)\right\} \times$
$\times \left(r^{(-1)}\left(\frac{\lambda}{p(1-p)} \int\limits_0^{\beta p^2} \frac{2r(N_{B_t}(\sigma^{(-1)}(u)))}{(H_{B_t}(\sigma^{(-1)}(u)))^{1/2}} du\right)\right)^2$ and $W_1(\lambda, t, p) = \left(N_{B_t}\left(\sigma^{(-1)}(\beta p)\right)\right)^{\frac{1}{v}} \times$
$\times \inf_{v \geqslant \frac{1}{1-p}} \min\left\{\left(\int\limits_1^{N_{B_t}(\sigma^{(-1)}(\beta p))+1} \exp\left\{\left(v\lambda\sigma\left(2x\sigma^{(-1)}(\beta p)\right)\right)^2/2 + v\lambda\delta\left(2x\sigma^{(-1)}(\beta p)\right)\right\} dx\right)^{\frac{1}{v}} \times\right.$
$\left.\times \inf_{w > 1} \exp\left\{\frac{\psi(wv\lambda\gamma(t))}{wv} + \max_{u \in B_t}\left\{\frac{(w-1)\psi\left(\frac{wv}{w-1}\lambda\gamma(u)\right)}{wv} + \lambda f(u)\right\} - \lambda f(t)\right\}\right\}.$

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STUDY OF THE DYNAMICS OF THE INTEREST RATE SWAP USING MACHINE LEARNING METHODS

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For the European financial system, the interest rate swap is a well-known mechanism to reduce the potential effects of the risks of interest rates changes, but it is new to the Ukrainian interbank market. Swop assumes that one party (the National Bank of Ukraine) offers the other a floating interest rate while the other (the commercial bank) offers a fixed interest rate based on a conditional value. The floating rate is calculated based on the Ukrainian overnight interbank rate index (UONIA). Future cash flows are discounted at rates determined by the yield to maturity curve for UAH domestic government bonds. The parties agree on how to calculate the difference in interest payments within the predetermined period of time. The analysis of the fair value of this financial instrument at future points in time is the main concern of mathematical modeling of interest rate swap transactions. Predicting future changes in fair value is particularly crucial when yield to maturity curve for UAH domestic government bonds coefficients vary and when there are specific trends in the index of overnight interbank rates. The sensitivity of the specified factors to the interest rate swap's dynamics was investigated, forecast of the instrument's future dynamics based on the change in important macroeconomic indicators using machine learning methods was developed.

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ABSTRACTS

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